A string oscillations simulation with nonlinear conditions

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In this talk we consider the initial-boundary value problems describing oscillation processes with nonlinear conditions. Analogues of the d'Alembert formula are obtained. Some control problems are analysed and the explicit forms of control functions are presented.

Suppose a string is located along the segment $[0, l]$. Assume that the right end of the string moves along a vertical needle(without friction) inside a sleeve, representing by $[-h, h]$. Notice that we consider the case where the sleeve can move in perpendicular to the axis Ox direction and its movement is given by $C(t) = [-h, h] + \xi(t)$. The mathematical model of such problem can be described as

$$\begin{cases} \dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial^2 u}{\partial t^2}, \quad 0 < x < l,\ 0 < t < T, \\ u(x,0) = \varphi(x), \\ \dfrac{\partial u}{\partial t}(x,0) = 0, \\ u(0,t) = \mu(t), \quad u(l,t) \in C(t), \\ -u'_x(l,t) \in N_{C(t)}(u(l,t)), \end{cases}$$

where the set $N_{C(t)}(u(l,t))$ is the outward normal cone to $C(t)$ at $u(l,t)$.

Also we consider a problem on a geometrical graph

$$\begin{cases} \dfrac{\partial^2 u_i}{\partial x^2} = \dfrac{\partial^2 u_i}{\partial t^2}, \quad 0 < x < l,\ 0 < t < T\ (i = 1, 2, \dots, n), \\ u_i(x,0) = \varphi_i(x), \\ \dfrac{\partial u_i}{\partial t}(x,0) = 0, \quad -\displaystyle\sum_{i=1}^{n} \dfrac{\partial u_i}{\partial x}(l-0,t) \in N_{C(t)}(u(l,t)), \\ u(l,t) = u_1(l,t) = u_2(l,t) = \cdots = u_n(l,t), \quad u(l,t) \in C(t), \\ u_i(0,t) = \mu_i(t). \end{cases}$$

Acknowledgement

The research was supported by Russian Science Foundation grant 16-11-10125, performed in Voronezh State University.

2010 Mathematics Subject Classification: 34B45, 34H05, 34C55, 34B15.

References

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