Kurzweil-Stieltjes integral and its application to dynamic equations on time-scales

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The contribution is based on the joint research with Giselle Antunes Monteiro and Antonín Slavík. In particular, we will rely on our recent contributions concerning the continuous dependence on a parameter k of solutions to linear integral equations of the form

\[ x(t) = \tilde{x}_k + \int_a^t d[A_k]x + f_k(t) - f_k(a), \quad t \in [a, b], \; k \in \mathbb{N}, \]

where \(-\infty < a < b < \infty\), \(X\) is a Banach space, \(L(X)\) is the Banach space of linear bounded operators on \(X\), \(\tilde{x}_k \in X\), \(A_k: [a, b] \to L(X)\) have bounded variations on \([a, b]\), \(f_k: [a, b] \to X\) are regulated on \([a, b]\). The integrals are understood as the abstract Kurzweil-Stieltjes integrals and the studied equations are usually called Generalized Linear Differential Equations. In those our contributions we succeeded to get the continuous dependence result for the difficult case when the variations \(\text{var}_a^b A_k\) need not be uniformly bounded.

In a sense, it was inspired by the Opial’s result for ODEs from 1967. As a consequence and thanks to the relationship between the Kurzweil-Stieltjes integral and the time scale integral disclosed by A. Slavík we are able to present also new Opial type results concerning the continuous dependence on a parameter of solutions to dynamic equations on time scales.

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