Localized extrema of ground state solution for nonlinear Schrödinger equation with non-monotone potential

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On an arbitrary interval $[a, b]$, we give some conditions for the potentials: $\mu \in \mathbb{R}$ - chemical, $V(x)$ - non-monotone external and $f(x, s)$ - nonlinear, such that every non-negative solution $u = u(x)$, $x \in \mathbb{R}$, of the nonlinear Schrödinger equation:

$$u'' + \left(\mu - \frac{2m}{h^2}\, V(x)\right) u + \frac{2m}{h^2}\, f(x, |u|^2) u = 0, \tag{1}$$

has a local maximum in $[a, b]$: [there exists a point $x_* = x_*(u) \in [a, b]$ such that $u'(x_*) = 0$ and $u(x_*) > u(x)$ for all $x \in (x_* - \varepsilon, x_* + \varepsilon)$ and some $\varepsilon = \varepsilon(u) > 0$]. As a consequence, it follows:

Corollary. *Let $f(x, s) \geq -g(x)$, $s \geq 0$, $x \in \mathbb{R}$, where $g(x) \leq 0$ or $g(x) \equiv 0$ - the general attractive case of $f(x, s)$ and $g(x) \geq 0$, $g(x) \not\equiv 0$ - a special repulsive case of $f(x, s)$. If we suppose that*

$$\mu - \frac{2m}{h^2}\big(V(x) + g(x)\big) > \lambda_1 \quad \text{in } [a, b], \tag{2}$$

where λ_1 is the first eigenvalue of the Laplacian operator in (a, b): [$\varphi'' + \lambda_1 \varphi = 0$ in (a, b) for some $\varphi \in C_0([a, b]) \cap C^2(a, b)$], then every solution $u(x)$ of (1) has a stationary point $x^* \in [a, b]$.

Moreover, if $u(x) \geq 0$ in $[a, b]$ and $u(x)$ possesses at most finite number of zeros in $[a, b]$, then the point x^* is unique as well as $u(x)$ attains its local maximum at x^*.

If $g(x) \equiv 0$, then by (2) it takes for $[a, b]$ an interval where $V(x)$ attains its local minimum.

This talk is organized as follows: **1.** experimental verification of Bose-Enstein condensate - BEC, **2.** nonlinear Schrödinger equation as a mathematical model for BEC, **3.** numerical and exact solution verifications for the non-monotonic behaviour of particle density in BEC, **4.** mathematical proof for the non-monotonic behaviour of particle density in BEC and main results.

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