Non-monotone travelling waves solutions for a monostable reaction-diffusion equations with delay

Adrián Goméz

Concepción, Chile

In this talk, we consider a family of nonlinear delayed reaction-diffusion equations

$$u_t(t,x) = u_{xx}(t,x) - u(t,x) + g(u(t-h,x)), \tag{1}$$

with $h > 0$. In our case, the non-linear reaction term $g(x)$ satisfies the following conditions: (i) $g(0) = 0$, $g(\kappa) = \kappa$, for some $\kappa > 0$; (ii) $g'(0) >1$, $g'(\kappa) < 0$; (iii) $g(x) > 0$, $\forall\ x \in (0,\kappa)$ i.e. equation (1) has two constant solutions $u_0 \equiv 0$, $u_\kappa \equiv \kappa$.

A traveling wave solution of (1) is a positive solution $u(t,x) = \Phi(x+ct)$, where the *wave's shape* $\phi : \mathbb{R} \to \mathbb{R}$ satisfies $\phi(-\infty) = 0$, $\phi(+\infty) = \kappa$ and the constant $c > 0$ is called *wave's speed*.

For some specifics type of g (see [4, 1]) and for some parameters h, c, the shape of the traveling wave for (1) could be of the following different forms: (i) monotone increasing, (ii) eventually monotone, non-monotone(finite oscillations) (iii) slowly oscillating(infinity oscillations).

The family (1) includes some classical models from biology, intensively studied, such as Nicholson's Blowflies equation and Mackey-Glass equation where the three geometric possibilities for the wave's shape have been observed numerically or analytically in some cases (see [5, 4, 1]). It is remarkable that each of the geometric possibilities has a biological interpretation [3, 1] which makes interesting to know the existence of them. In this work, we analyze the existence of a eventually monotone, non-monotone traveling wave for the classical Nicholson's blowflies equation (that is, with $g(x) = \frac{p}{\delta}\, x\, \mathrm{e}^{-x}$ in (1)) for some values of parameters p, δ, h and c.

Acknowledgement

This research is supported by FONDECYT iniciacion 11130367.

2010 Mathematics Subject Classification: 39B99, 34K99, 35K57.

References

[1] A. Ivanov, C. Gomez and S. Trofimchuk, On the existence of non-monotone non oscillating wave fronts. J. Math. Anal. Appl. 419 (2014) 606–616.

[2] A. Gomez and S. Trofimchuk, Global continuation of monotone wavefronts. J. London Math. Soc. (2) 89 (2014) 47–68.

[3] S.A. Gourley, J.W.-H. So, and J.H. Wu, Nonlocality of reaction-diffusion equations induced by delay: Biological modeling and nonlinear dynamics. J. Math. Sci. 124 (2004) 5119–5153.

[4] E. Trofimchuk, V. Tkachenko and S. Trofimchuk, Slowly oscillating wave solutions of a single species reaction-diffusion equation with delay, J. Diff. Eq. 245 (2008) 2307–2332.

[5] D. Liang and J. Wu, Travelling waves and numerical approximations in a reaction advection diffusion equation with nonlocal delayed effects. J. Nonlinear Sci. 13 (2003) 289–310.