## Measure functional differential equations with infinite time-dependent delay

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Measure functional differential equations ( in short **MFDEs** ) with finite delay of type

$$y(t) = y(t_0) + \int_{t_0}^{t} f(y_s, s)dg(s), \quad t \in [t_0, t_0 + \sigma], \tag{1}$$

have been introduced by Ferderson, Mesquita and Slavik in [1]. Here $y$ and $f$ are functions with values in $\mathbb{R}^n$, the integral on the right-hand side of (1) is the Kurzweil-Henstock integral with respect to a nondecreasing function $g$ and $y_s$ represents the "history" of $y$ at $s$. They showed that functional dynamic equations on time scales represent a special case of **MFDEs**, and they obtained results on the existence and uniqueness of solutions using the theory of generalized ordinary differential equations, which were introduced by J. Kurzweil in 1957 [4]. The case when the equation (1) is considered with infinite delay were later studied by A. Slavik in [5]. He described axiomatically a suitable phase space similarly as classical functional differential equations with infinite delay (see e.g. [2], [3] ), and he obtained results of existence and uniqueness.

We focus our attention on the equation (1) with infinite time-dependent delay, that means, we are interested to study the equation

$$y(t) = y(t_0) + \int_{t_0}^{t} f(y_{r(s)}, s)dg(s), \quad t \in [t_0, t_0 + \sigma], \tag{2}$$

where $r$ is a nondecreasing function such that $r(s) \leq s$, for all $s \in Dom(r)$.

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## References

[1] Federson M., Mesquita J. G., Slavik A. Measure functional differential equations and functional dynamics equations on time scales, *J. Diff. Equations*, **252**, 3816–3847 (2012).

[2] Hale J. K., Kato J. Phase space for retarded equations with infinite delay, *Funkcial. Ekvac.*, **21**, 11–41 (1978).

[3] Hino Y., Murakami S., Naito T. Funcional Differential Equations with Infinite Delay, Springer-Verlag, Berlin (1991).

[4] Kurzweil J. Generalized ordinary differential equations and continuous dependence on a parameter, *Czech. Math. J.*, **7(82)**, 418–448 (1957).

[5] Slavik A. Measure functional differential equations with infinite delay, *Nonlinear Analysis*, **79**, 140–155 (2013).