Convergence to a new profile of travelling front in Fisher’s population genetics model

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We consider the semilinear Fisher-Kolmogorov-Petrovski-Piscounov equation for the advance of an advantageous gene in biology:

\[
\begin{cases}
\frac{\partial u}{\partial t} - \frac{\partial^2 u}{\partial x^2} = f(u) & \text{for } (x, t) \in \mathbb{R} \times \mathbb{R}_+; \\
u(x, 0) = u_0(x) & \text{for } x \in \mathbb{R}.
\end{cases}
\]

In contrast with previous works on this topic, we relax the differentiability hypothesis on \( f \) to being only Hölder-continuous and “one-sided” Lipschitz-continuous (i.e., \( s \mapsto f(s) - Ls: \mathbb{R} \to \mathbb{R} \) is monotone decreasing, for some constant \( L \in \mathbb{R}_+ \)). In particular, our hypotheses allow for the singular derivatives

\[
f'(0) = \lim_{s \to 0} \frac{f(s)}{s} = -\infty \quad \text{and} \quad f'(1) = \lim_{s \to 1} \frac{f(s)}{s - 1} = -\infty.
\]

This type of reaction function \( f \) has been studied extensively in biological models of various kinds of generalized logistic growth.

The fact that reaction function \( f \) is not smooth allows for the introduction of travelling waves with a new profile. We study existence and uniqueness of this new profile, as well as a long-time asymptotic behavior of the solutions of the Cauchy problem to a travelling wave with this profile. Presented results are based on joint paper with P. Takáč entitled “Convergence to travelling waves in Fisher’s population genetics model with a non-Lipschitzian reaction term” and published online in *J. Math. Biol.* on February 14, 2017.

**2010 Mathematics Subject Classification:** 35Q92, 92D10, 35K91, 35K58.