The local minimum theorem and the mountain pass theorem: complements, relationships, remarks and applications

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The existence of at least two or three non-zero solutions for nonlinear elliptic eigenvalue problems is established. The basic ingredients are the local minimum theorem obtained in [6] and the classical Ambrosetti-Rabinowitz theorem (see [1]). Some remarks on the mountain pass theorem and its relationships with the local minima are highlighted (see [7]); further, a note on parameters for which the above problems admit solutions is done (see [2]) as well as a qualitative property of the obtained local minimum is investigated (see [3]). By an appropriate combination of previous results, theorems of two and three critical points are obtained (see [8, 5]) and a variant of the three critical points theorem, where the classical compactness condition of Palais-Smale is not assumed, is also emphasized (see [4]).

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