On asymptotic properties of blow-up and Kneser solutions to higher-order
Emden-Fowler type differential equations

Irina Astashova
Moscow, Russia

Consider the equation
\[ y^{(n)} = p(x, y, y', \ldots, y^{(n-1)}) |y|^k \text{sign } y, \quad n > 4, \ k > 1. \] \tag{1}

A new result is proved on asymptotic behavior of blow-up and Kneser (see [1, Definition 13.1])
solutions to this equation.

**Theorem 1.** Suppose \( p \in C(R^{n+1}) \cap \text{Lip}_{y_0, \ldots, y_{n-1}}(R^n) \) and \( p \to p_0 > 0 \) as \( x \to x^*, y_0 \to \infty, \ldots, y_{n-1} \to \infty \). Then for any integer \( n > 4 \) there exists \( K > 1 \) such that for any real \( k \in (1, K) \),
any solution to equation (1) tending to \( +\infty \) as \( x \to x^* - 0 \) has power-law asymptotic behavior,
namely \( y(x) = C(x^* - x)^{-\alpha}(1 + o(1)) \) with
\[ \alpha = \frac{n}{k - 1}, \quad C^{k-1} = \frac{1}{p_0} \prod_{j=0}^{n-1} (j + \alpha). \] \tag{2}

**Theorem 2.** Suppose \( p \in C(R^{n+1}) \cap \text{Lip}_{y_0, \ldots, y_{n-1}}(R^n) \) and \((-1)^n \ p \to p_0 > 0 \) as \( x \to \infty, \ y_0 \to 0, \ldots, y_{n-1} \to 0 \). Then for any integer \( n > 4 \) there exists \( K > 1 \) such that all Kneser solutions
to equation (1) with any real \( k \in (1, K) \) tend to zero with power-law asymptotic behavior, namely
\( y(x) = C(x - x^*)^{-\alpha}(1 + o(1)), \ x \to \infty, \) with some \( x^* \) and \( \alpha, \ C \) given by (2).

Earlier it was proved that for \( n = 3, 4 \) all blow-up and Kneser solutions to equation (1) have
the power-law asymptotic behavior (see [2]). It was also proved for equation (1) with \((-1)^n \ p \equiv p_0 > 0 \)
for sufficiently large \( n \) (see [3]) and for \( n = 12, 13, 14 \) (see [4]) that there exists \( k > 1 \) such that
equation (1) has a solution with non-power-law behavior, namely \( y(x) = (x - x^*)^{-\alpha} h(\log (x - x^*)), \)
where \( h \) is a positive periodic non-constant function on \( R \). For blow-up solutions see also [4, 5].

2010 Mathematics Subject Classification: 34C99.

References


Russian), in: I. V. Astashova (ed.), Qualitative Properties of Solutions to Differential Equations and

37 (1999), No 2, 305–322.

[4] I. Astashova. On power and non-power asymptotic behavior of positive solutions to Emden-Fowler type
1–15.