

On effective asymptotic formulas for two-dimensional wave equation with localized right-hand side

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We consider the Cauchy problem for the two-dimensional non-homogeneous wave equation with variable coefficients

$$\frac{\partial^2 \eta}{\partial t^2} - \nabla c^2(x) \nabla \eta = \frac{\partial g(t)}{\partial t} V \left(\frac{x}{\mu} \right) \quad \eta|_{t=0} = 0, \quad \eta_t|_{t=0} = 0, \quad t \geq 0, \quad x \in \mathbb{R}_x^2, \quad (1)$$

where the function $g(t)$ is either (a) Dirac delta function $\delta(t)$ or (b) some smooth “delta-like” function. Here $V(y)$ is smooth in \mathbb{R}_y^2 , decaying faster than $1/|y|_1^{\kappa_1}$ (where $\kappa_1 > 2$) as $|y| \rightarrow \infty$. Function $g(t)$ in case (b) is smooth on $[0, \infty)$, decaying faster than $1/t^{\kappa_2}$ (where $\kappa_2 > 1$) as $t \rightarrow \infty$, moreover, $g(t) = 0$ $t \leq 0$.

This problem describes, in the linear approximation, tsunami waves appearing due to local bottom displacements (see e.g. [1, 2]). Usually, the size of the source, i.e. the domain, where these local displacements take place is much less than the size of the ocean. Their ratio μ , may be considered as a small parameter and used in the asymptotic analysis of the solution.

The case (a) describes the so-called ‘piston model’, in which the source appears and disappears simultaneously. In the case (b) the source is time-spread, i.e. its action has small but finite duration. The model (b) seems to be better than (a) from the physical viewpoint.

In [3], the asymptotics for the solution in case (a) was found with use of Maslov’s canonical operator [4]. We show that this solution can be used to construct approximations to the solution in case (b). More precisely, the latter solution is obtained by applying some differential operator to the solution in case (a). We present a series of numerical experiments, showing that the approximation is quite accurate.

Acknowledgement

This work was supported by Russian Science Foundation (grant no. 16-11-10282).

2010 Mathematics Subject Classification: 58J37.

References

- [1] C. C. Mei, *Applied Dynamics of Ocean Surface Waves* (World Sci., Singapore, 1989).
- [2] E. N. Pelinovskii, *Hydrodynamics of Tsunami Waves* (IPF RAN, Nizhnii Novgorod, 1996).
- [3] S. Dobrokhotov, D. Minenkov, V. Nazaikinskii, and B. Tirozzi, Operator theory, pseudo-differential equations, and mathematical physics, 95–125, *Oper. Theory Adv. Appl.*, 228, Birkhuser/Springer Basel AG, Basel, 2013.
- [4] V. P. Maslov and M. V. Fedoryuk, *Semiclassical Approximation for Equations of Quantum Mechanics* (Nauka, Moscow, 1976)