Positive radial solutions for systems involving potential Lane-Emden nonlinearities and Minkowski operator

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Introduction

Let Ω be a bounded domain in $\mathbb{R}^N$ $(N \geq 2)$ with boundary $\partial\Omega$ of class C^2.

$$\begin{cases} \mathcal{M}(u) + \lambda F_u(x,u,v) = 0, & x \in \Omega, \\ \mathcal{M}(v) + \lambda F_v(x,u,v) = 0, & x \in \Omega, \\ u|_{\partial\Omega} = 0 = v|_{\partial\Omega}, \end{cases} \tag{1}$$

with $F : \Omega \times \mathbb{R}^2 \to \mathbb{R}$ satisfying:

(H_F) (i) $F(\cdot, u, v) : \Omega \to \mathbb{R}$ is measurable for all $(u,v) \in \mathbb{R}^2$ and $F(\cdot, 0, 0) = 0$;

(ii) $F(x, \cdot, \cdot) : \mathbb{R}^2 \to \mathbb{R}$ is of class C^1 on $\mathbb{R}^2$ for a.e. $x \in \Omega$;

(iii) for each $\rho > 0$ there is some $\alpha_\rho \in L^\infty(\Omega)$ such that

$$|\nabla F(x,u,v)| \leq \alpha_\rho(x) \text{ for a.e. } x \in \Omega,\ \forall\ (u,v) \in \mathbb{R}^2 \text{ with } |(u,v)| \leq \rho,$$

• By a **solution** of (1) we mean a couple of functions $(u,v) \in W^{2,p}(\Omega) \times W^{2,q}(\Omega)$ with some $p, q > N$, such that $\|\nabla u\|_\infty < 1$, $\|\nabla v\|_\infty < 1$, which satisfies the equations a.e. in Ω and vanishes on $\partial\Omega$.

Theorem 1.

Assume (H_F) *and that*

$$\begin{aligned} F(x,0,v) = F_u(x,0,v) = F_u(x,u,0) = 0 \quad \textit{and} \\ F(x,u,0) = F_v(x,u,0) = F_v(x,0,v) = 0, \end{aligned} \tag{2}$$

for a.e. $x \in \Omega$ *and all* $(u,v) \in [0,\infty)^2$.

If the following hold true:

(iv) $\exists\ R_1 > 0 : \begin{cases} F_u(x,u,v) > \ (\geq)\ 0 \\ F_v(x,u,v) \geq \ (>)\ 0 \end{cases}$, *for a.e.* $x \in \Omega,\ \forall\ u,v \in (0,R_1)$;

(v) $\displaystyle\lim_{|(u,v)|\to 0} \frac{F(x,u,v)}{|(u,v)|^2} = 0$ *uniformly with* $x \in \Omega$,

then there exists $\Lambda > 0$ *s.t. for all* $\lambda > \Lambda$ *problem* (1) *has at least* **two** *solutions with each component* **nontrivial** *(and non-negative).*

Proof. Set $\widetilde{F}(x,u,v) = F(x,u_+,v_+)$ $(x \in \Omega,\ u,v \in \mathbb{R})$ and consider

$$\begin{cases} \mathcal{M}(u) + \lambda \widetilde{F}_u(x,u,v) = 0, & x \in \Omega, \\ \mathcal{M}(v) + \lambda \widetilde{F}_v(x,u,v) = 0, & x \in \Omega, \\ u|_{\partial\Omega} = 0 = v|_{\partial\Omega} \end{cases} \tag{3}$$

- (u,v) solution for (3) $\Rightarrow$ (u,v) has non-negative components and solves (1).

$$K_0 := \{u \in W^{1,\infty}(\Omega) : \|\nabla u\|_\infty \leq 1,\ u|_{\partial\Omega} = 0\}$$

$$\Psi(u) = \begin{cases} \displaystyle\int_\Omega [1 - \sqrt{1 - |\nabla u|^2}] & \text{for} \quad u \in K_0 \\ +\infty & \text{for} \quad u \in C(\overline{\Omega}) \setminus K_0 \end{cases}$$

$*$ Ψ is convex and lower semicontinuous on $C(\overline{\Omega})$

$$\widetilde{\mathfrak{F}}(u, v) = \int_\Omega \widetilde{F}(x, u, v)$$

$*$ $\widetilde{\mathfrak{F}}$ is of class C^1 on $C(\overline{\Omega}) \times C(\overline{\Omega})$

$$I_\lambda(u,v) := \Psi(u) + \Psi(v) - \lambda\widetilde{\mathfrak{F}}(u,v), \qquad \forall (u,v) \in C(\overline{\Omega}) \times C(\overline{\Omega})$$

• (u,v) critical point of I_λ (in the sense of Szulkin) $\Rightarrow$ (u,v) solution of (3)

• $\exists\, \Lambda > 0$ s.t. $\forall \lambda > \Lambda$:

(*a*) I_λ has a negative minimum,

(*b*) I_λ has a positive value at a (mountain pass) critical point

$\Rightarrow$ I_λ has two nontrivial critical points; each such a critical point is a solution of (3) *having each component nontrivial*.

Example 2.

There exists $\Lambda > 0$ s.t., for all $\lambda > \Lambda$, the system

$$\begin{cases} \mathcal{M}(u) + \lambda u v^2 = 0, & x \in \Omega, \\ \mathcal{M}(v) + \lambda u^2 v = 0, & x \in \Omega, \\ u|_{\partial\Omega} = 0 = v|_{\partial\Omega} \end{cases} \tag{4}$$

has at least two solutions with each component nontrivial and non-negative.

- *take $F(x,u,v) = u^2 v^2 / 2$*

More general: • $F(x,u,v) = \mu(|x|)u^{p+1}v^{q+1}$ under the hypothesis:

(*H*) *The non-negative exponents* p, q *satisfy* $\max\{p,q\} > 1$ *and the function* $\mu : [0,R] \to [0,\infty)$ *is continuous and* $\mu(r) > 0$ *for all* $r \in (0,R]$.

$$\begin{cases} \mathcal{M}(\mathrm{u}) + \lambda\mu(|x|)(p+1)\mathrm{u}^p\mathrm{v}^{q+1} = 0, & x \in \Omega, \\ \mathcal{M}(\mathrm{v}) + \lambda\mu(|x|)(q+1)\mathrm{u}^{p+1}\mathrm{v}^{q} = 0, & x \in \Omega, \\ u|_{\partial\Omega} = 0 = v|_{\partial\Omega} \end{cases} \tag{5}$$

Theorem 1 $\Rightarrow \exists\ \Lambda > 0$ s.t. $\forall\lambda > \Lambda$ the system (5) has at least two solutions with each component nontrivial (and non-negative).

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In the case on a single equation in a ball:

$$\mathcal{M}(\mathrm{u}) + \lambda\mu(|x|)\mathrm{u}^{\alpha} = 0 \ \text{ in } \mathcal{B}(R), \ \ \mathrm{u}|_{\partial\mathcal{B}(R)} = 0 \ \ (\alpha > 1)$$

a **sharper result** holds true: there exists $\Lambda > 0$ s.t. it has zero, at least one or at least two positive solutions according to $\lambda \in (0, \Lambda)$, $\lambda = \Lambda$ or $\lambda > \Lambda$.

• • $r := |x|$ and $\mathrm{u}(x) = u(r)$, $\mathrm{v}(x) = v(r)$, the *Dirichlet* problem (5) in $\Omega = \mathcal{B}(R)$ reduces to the *mixed* boundary value problem:

$$\begin{cases} [r^{N-1}\varphi(u')]' + \lambda r^{N-1}\mu(r)(p+1)u^p v^{q+1} = 0, \\ [r^{N-1}\varphi(v')]' + \lambda r^{N-1}\mu(r)(q+1)u^{p+1} v^{q} = 0, \\ u'(0) = u(R) = 0 = v(R) = v'(0), \end{cases} \tag{6}$$

where

$$\varphi(y) = \frac{y}{\sqrt{1-y^2}} \quad (y \in \mathbb{R}, \ |y| < 1).$$

Lower and upper solutions

Consider the general system:

$$\begin{cases} \left[r^{N-1}\varphi(u')\right]' + r^{N-1}f_1(r,u,v) = 0, \\ \left[r^{N-1}\varphi(v')\right]' + r^{N-1}f_2(r,u,v) = 0, \\ u'(0) = u(R) = 0 = v(R) = v'(0), \end{cases} \tag{7}$$

where $f_1, f_2 : [0,R] \times \mathbb{R}^2 \to \mathbb{R}$ are continuous.

By a *solution* of (7) we mean a couple of functions $(u,v) \in C^1[0,R] \times C^1[0,R]$ with $||u'||_\infty < 1$, $||v'||_\infty < 1$ and $r \mapsto r^{N-1}\varphi(u'(r))$, $r \mapsto r^{N-1}\varphi(v'(r))$ of class C^1 on $[0,R]$, which satisfies problem (7). Here and below, we denote by $\|\cdot\|_\infty$ the usual sup-norm on $C := C[0,R]$. We say that $u \in C$ is *positive* if $u > 0$ on $[0,R)$. By a *positive solution* of (7) we understand a solution (u,v) with both u and v positive.

A *lower solution* of (7) is a couple of functions $(\alpha_u, \alpha_v) \in C^1 \times C^1$, s.t. $\|\alpha_u'\|_\infty < 1$, $\|\alpha_v'\|_\infty < 1$, the mappings $r \mapsto r^{N-1}\varphi(\alpha_u'(r))$, $r \mapsto r^{N-1}\varphi(\alpha_v'(r))$ are of class C^1 on $[0, R]$ and satisfies

$$\begin{cases} [r^{N-1}\varphi(\alpha_u')]' + r^{N-1} f_1(r, \alpha_u, \alpha_v) \geq 0, \\ [r^{N-1}\varphi(\alpha_v')]' + r^{N-1} f_2(r, \alpha_u, \alpha_v) \geq 0, \\ \alpha_u(R) \leq 0, \quad \alpha_v(R) \leq 0. \end{cases}$$

An *upper solution* $(\beta_u, \beta_v) \in C^1 \times C^1$ is defined by reversing the above inequalities.

• $J_1, J_2 \subset \mathbb{R}$. In the terminology of [14], a function $f = f(r,s,t) : [0,R] \times J_1 \times J_2 \to \mathbb{R}$ is said to be *quasi-monotone nondecreasing* with respect to t (resp. s) if for fixed r, s (resp. r, t) one has

$$f(r,s,t_1) \le f(r,s,t_2) \text{ as } t_1 \le t_2 \quad (\text{resp. } f(r,s_1,t) \le f(r,s_2,t) \text{ as } s_1 \le s_2).$$

Proposition 2.1.

If (7) has a lower solution (α_u, α_v) and an upper solution (β_u, β_v) s.t. $\alpha_u(r) \le \beta_u(r)$, $\alpha_v(r) \le \beta_v(r)$ for all $r \in [0,R]$ and $f_1(r,s,t)$ (resp. $f_2(r,s,t)$) is quasi-monotone nondecreasing with respect to t (resp. s), then (7) has a solution (u,v) s.t. $\alpha_u(r) \le u(r) \le \beta_u(r)$ and $\alpha_v(r) \le v(r) \le \beta_v(r)$ for all $r \in [0,R]$.

$C^1 := C^1[0,R]$ with $||u||_1 = ||u||_\infty + ||u'||_\infty$

$C^1 \times C^1$ with $||(u,v)|| = \max\{||u||_\infty, ||v||_\infty\} + \max\{||u'||_\infty, ||v'||_\infty\}$

$$\mathcal{C}^1_M = \{(u,v) \in C^1 \times C^1 : u'(0) = u(R) = 0 = v(R) = v'(0)\}$$

N_{f_i} =the Nemytskii operator associated to f_i $(i = 1,2)$, i.e.,

$$N_{f_i} : C \times C \to C,\ N_{f_i}(u,v) = f_i(\cdot, u(\cdot), v(\cdot)) \quad (u, v \in C),$$

$$S : C \to C,\ Su(r) = \frac{1}{r^{N-1}} \int_0^r t^{N-1} u(t)dt \quad (r \in [0,R]),\ Su(0) = 0;$$

$$K : C \to C^1,\ Ku(r) = \int_r^R u(t)dt \quad (r \in [0,R]).$$

Proposition 2.2.

A couple of functions $(u, v) \in \mathcal{C}^1_M$ *is a solution of* (7) *if and only if it is a fixed point of the compact nonlinear operator*

$$\mathcal{N}_f : \mathcal{C}^1_M \to \mathcal{C}^1_M, \quad \mathcal{N}_f = \left(K \circ \varphi^{-1} \circ S \circ N_{f_1}, K \circ \varphi^{-1} \circ S \circ N_{f_2}\right).$$

In addition, any fixed point (u, v) *of* $\mathcal{N}_f$ *satisfies*

$$\|u'\|_\infty < 1, \quad \|v'\|_\infty < 1, \quad \|u\|_\infty < R, \quad \|v\|_\infty < R, \tag{8}$$

and

$$d_{LS}[I - \mathcal{N}_f, B_\rho, 0] = 1 \text{ for all } \rho \geq R + 1.$$

In particular, problem (7) *has at least one solution in* B_ρ *for all* $\rho \geq R + 1$.

- When system (7) is potential:

$$\begin{cases} [r^{N-1}\varphi(u')]' = r^{N-1}F_u(r,u,v), \\ [r^{N-1}\varphi(v')]' = r^{N-1}F_v(r,u,v), \\ u'(0) = u(R) = 0 = v(R) = v'(0), \end{cases} \tag{9}$$

with $F = F(r,u,v) : [0,R] \times \mathbb{R}^2 \to \mathbb{R}$ continuous, s.t. F_u and F_v exist and are continuous on $[0,R] \times \mathbb{R}^2$, then a variational approach is available:

$$K_0 = \{u \in W^{1,\infty}[0,R] : \|u'\|_\infty \leq 1, u(R) = 0\}.$$

$$\psi(u) = \begin{cases} \displaystyle\int_0^R r^{N-1}[1 - \sqrt{1 - u'^2}]dr & \text{for} \quad u \in K_0 \\ +\infty & \text{for} \quad u \in C \setminus K_0, \end{cases}$$

$$\Psi(u,v) := \psi(u) + \psi(v), \text{ for all } (u,v) \in C \times C.$$

$*$ Ψ is proper, convex and lower semicontinuous.

$$\mathcal{F}(u,v) := \int_0^R r^{N-1} F(r,u,v), \ (u, v \in C)$$

$*$ $\mathcal{F}$ is of class C^1 on $C \times C$

$$\mathcal{I} := \Psi + \mathcal{F}$$

Proposition 2.3.

If $(u, v) \in C \times C$ is a critical point of $\mathcal{I}$ (in the sense of Szulkin), then it is a solution of system (9). Moreover, system (9) has a solution which is a minimum point of $\mathcal{I}$ on $C \times C$.

Lemma 3.

Assume that (7) has a lower solution (α_u, α_v) and an upper solution (β_u, β_v) s.t. $\alpha_u(r) \leq \beta_u(r)$, $\alpha_v(r) \leq \beta_v(r)$ for all $r \in [0, R]$ and $f_1(r, s, t)$ (resp. $f_2(r, s, t)$) is quasi-monotone nondecreasing with respect to t (resp. s). Let

$$\mathcal{A}_{\alpha,\beta} := \{(u, v) \in \mathcal{C}^1_M : \alpha_u \leq u \leq \beta_u,\ \alpha_v \leq v \leq \beta_v\}.$$

Assume also that (7) has an unique solution (u_0, v_0) in $\mathcal{A}_{\alpha,\beta}$ and there exists $\rho_0 > 0$ s.t. $\overline{B}((u_0, v_0), \rho_0) \subset \mathcal{A}_{\alpha,\beta}$. Then

$$d_{LS}[I - \mathcal{N}_f, B((u_0, v_0), \rho), 0] = 1, \quad \text{for all } 0 < \rho \leq \rho_0,$$

where $\mathcal{N}_f$ stands for the fixed point operator associated to (7).

- $g_1, g_2 : [0, R] \times [0, \infty)^2 \to [0, \infty)$ continuous

$$\begin{cases} [r^{N-1}\varphi(u')]' + r^{N-1} g_1(r, u_+, v_+) = 0, \\ [r^{N-1}\varphi(v')]' + r^{N-1} g_2(r, u_+, v_+) = 0, \\ u'(0) = u(R) = 0 = v(R) = v'(0), \end{cases} \tag{10}$$

where $\xi_+ := \max\{0, \xi\}$.

Lemma 4.

Assume that g_1, g_2 *satisfy hypothesis:*

(H_g) (i) $g_1(r,s,t) > 0 < g_2(r,s,t), \ \forall s,t > 0, \ \forall r \in (0,R]$;

(ii) $g_1(r,\xi,0) = g_2(r,0,\xi) = 0, \ \forall \xi > 0, \ \forall r \in (0,R]$.

If there is some $M > 0$ *s.t. either*

$$\lim_{s \to 0_+} \frac{g_1(r,s,t)}{s} = 0 \text{ uniformly with } r \in [0,R], \ t \in [0,M] \quad (11)$$

or

$$\lim_{t \to 0_+} \frac{g_2(r,s,t)}{t} = 0 \text{ uniformly with } r \in [0,R], \ s \in [0,M], \quad (12)$$

then there exists $\rho_0 > 0$ *s.t.*

$$d_{LS}[I - \mathcal{N}_g, B_\rho, 0] = 1 \text{ for all } 0 < \rho \le \rho_0,$$

where $\mathcal{N}_g$ *is the fixed point operator associated to problem* (10).

Remark 2.1.

Under hypothesis (H_g) in Lemma 4 any nontrivial solution of problem (10) is a positive solution of the system

$$\begin{cases} [r^{N-1}\varphi(u')]' + r^{N-1}g_1(r,u,v) = 0, \\ [r^{N-1}\varphi(v')]' + r^{N-1}g_2(r,u,v) = 0, \\ u'(0) = u(R) = 0 = v(R) = v'(0). \end{cases} \tag{13}$$

Back to the gradient system (6) under hypothesis (H)

Theorem 5.

Assume (H). Then there exists $\Lambda > 0$ s.t. the system (6) has zero, at least one or at least two positive solutions according to $\lambda \in (0, \Lambda)$, $\lambda = \Lambda$ or $\lambda > \Lambda$.

Proof. We assume that $0 < q \leq p > 1$ and we make use of the equivalent system:

$$\begin{cases} [r^{N-1}\varphi(u')]' + \lambda r^{N-1}\mu(r)(p+1)u_+^p v_+^{q+1} = 0, \\ [r^{N-1}\varphi(v')]' + \lambda r^{N-1}\mu(r)(q+1)u_+^{p+1} v_+^q = 0, \\ u'(0) = u(R) = 0 = v(R) = v'(0) \end{cases} \tag{14}$$

$$\mathcal{J}_\lambda(u, v) = \frac{2R^N}{N} - \int_0^R r^{N-1}[\sqrt{1-u'^2} + \sqrt{1-v'^2}]dr - \lambda \int_0^R r^{N-1}\mu(r)u_+^{p+1}v_+^{q+1}dr$$

$$u_0(r) = v_0(r) = R - r \Rightarrow \mathcal{J}_\lambda(u_0, v_0) < 0, \text{ for } \lambda > 0 \textbf{ large enough}$$

$$\Rightarrow \mathcal{S} := \{\lambda > 0 : (6) \text{ has a positive solution}\} \neq \emptyset$$

1. Existence of Λ; the cases $\lambda \in (0, \Lambda)$ and $\lambda = \Lambda$

- $\lambda \in \mathcal{S} \Rightarrow \lambda > 2N/[(p+1)R^{p+q+2} \max_{[0,R]} \mu](> 0)$

$$(0 <)\ \Lambda := \inf \mathcal{S}\ (< +\infty)$$

- $\Lambda \in \mathcal{S}$

2. The case $\lambda > \Lambda$*.*

• $(\Lambda, \infty) \subset \mathcal{S}$: $\lambda_0 \in (\Lambda, \infty) \overset{?}{\Rightarrow} \lambda_0 \in \mathcal{S}$

$>>$ (u_Λ, v_Λ) a positive solution of (6) with $\lambda = \Lambda \Rightarrow (u_\Lambda, v_\Lambda)$ is a lower solution for (14) with $\lambda = \lambda_0 >>$ an upper solution (u_{H_1}, v_{H_2}) for (14) with $\lambda = \lambda_0$ can be constructed s.t. $u_\Lambda < u_{H_1}$ $v_\Lambda < v_{H_2}$

$\Rightarrow$ (14) has a positive solution (Proposition 2.1) $\Rightarrow \lambda_0 \in \mathcal{S}$.

- $\lambda_0 \in (\Lambda, \infty) \overset{?}{\Rightarrow}$ (14) with $\lambda = \lambda_0$ has a second positive solution.

$>>$ (u_Λ, v_Λ) be the lower solution and (u_{H_1}, v_{H_2}) be the upper solution constructed as above

$>>$ fix (u_0, v_0) a positive solution of (14) with $\lambda = \lambda_0$ s.t.

$$(u_0, v_0) \in \mathcal{A} := \{(u, v) \in \mathcal{C}^1_M : u_\Lambda \leq u \leq u_{H_1},\ v_\Lambda \leq v \leq v_{H_2}\}.$$

$>> \exists\, \varepsilon > 0$ s.t. $\overline{B}((u_0, v_0), \varepsilon) \subset \mathcal{A}$

If (14) has a second solution contained in $\mathcal{A}$, then it is nontrivial and the proof is complete

⊙ If not, Lemma 3 $\Rightarrow$

$$d_{LS}[I - \mathcal{N}_{\lambda_0}, B((u_0, v_0), \rho), 0] = 1 \text{ for all } 0 < \rho \leq \varepsilon,$$

where $\mathcal{N}_{\lambda_0}$ is the fixed point operator associated to (14) with $\lambda = \lambda_0$

⊙ $d_{LS}[I - \mathcal{N}_{\lambda_0}, B_\rho, 0] = 1$ for all $\rho \geq R + 1$ (Proposition 2.2)

⊙ $d_{LS}[I - \mathcal{N}_{\lambda_0}, B_\rho, 0] = 1$ for $\rho > 0$ small (Lemma 4)

$\triangleright$ $\rho_1, \rho_2 > 0$ be small and $\rho_3 \geq R + 1$ s.t.

$$\bar{B}((u_0, v_0), \rho_1) \cap \bar{B}_{\rho_2} = \emptyset \text{ and } \bar{B}((u_0, v_0), \rho_1) \cup \bar{B}_{\rho_2} \subset B_{\rho_3}$$

Additivity-excision property of Leray-Schauder degree $\Rightarrow$

$$d_{LS}[I - \mathcal{N}_{\lambda_0}, B_{\rho_3} \backslash [\bar{B}((u_0, v_0), \rho_1) \cup \bar{B}_{\rho_2}], 0] = -1.$$

$\Rightarrow \mathcal{N}_{\lambda_0}$ has a fixed point $(u, v) \in B_{\rho_3} \backslash [\bar{B}((u_0, v_0), \rho_1) \cup \bar{B}_{\rho_2}] \Rightarrow$ (14) has a second positive solution.

Corollary 6.

Assume (H). Then there exists $\Lambda > 0$ s.t. the problem

$$\begin{cases} \mathcal{M}(\mathrm{u}) + \lambda\mu(|x|)(p+1)\mathrm{u}^p\mathrm{v}^{q+1} = 0 & \text{in } \mathcal{B}(R), \\ \mathcal{M}(\mathrm{v}) + \lambda\mu(|x|)(q+1)\mathrm{u}^{p+1}\mathrm{v}^q = 0 & \text{in } \mathcal{B}(R), \\ \mathrm{u}|_{\partial\mathcal{B}(R)} = 0 = \mathrm{v}|_{\partial\mathcal{B}(R)} \end{cases}$$

has zero, at least one or at least two positive solutions according to $\lambda \in (0, \Lambda)$, $\lambda = \Lambda$ or $\lambda > \Lambda$.

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Thank you for your attention!