Cycles in models of monetary and fiscal stabilization policies

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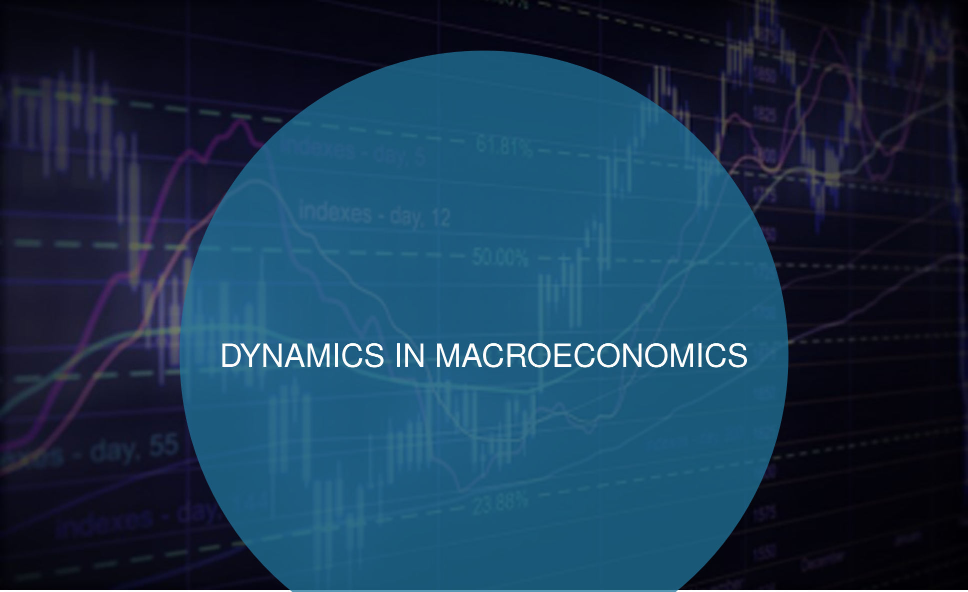

DYNAMICS IN MACROECONOMICS

THE MAIN GOAL

is to establish whether *stable* economical cycles are present in a class of models with flexible prices and mixture of fiscal and monetary policies.

METHOD USED

We focus on transition between stable steady-state to unstable one with at least one stable limit cycle modeling business-cycles. So Hopf-Andronov theory is used.

Introduction to model - used dynamical variables

We consider six-dimensional dynamical system with:

- *d* - private debt/capital ratio
- *y* - output/capital ratio
- π^e - expected rate of prices inflation
- ρ - nominal rate of interest of government bond
- ν - government expenditures/capital ratio
- *b* - government bond/capital ratio

The Equations (V1)

$$
\begin{aligned}
\dot{d} &= \Phi(g) - s_f[r - i(\rho, d)d] - (g + \pi)d \equiv F_1, \\
\dot{y} &= \alpha[c + \Phi(g) + \nu - y] \equiv \alpha F_2, \ \alpha > 0 \\
\dot{\pi}^e &= \gamma[\xi(\bar{\pi} - \pi^e) + (1 - \xi)(\pi - \pi^e)] \equiv F_3, \\
\dot{\rho} &= \beta_1(\pi - \bar{\pi}) + \beta_2(y - \bar{y}) \equiv F_4, \\
\dot{\nu} &= \sigma[\theta(\bar{y} - y) + (1 - \theta)(\bar{b} - b)] \equiv F_5, \\
\dot{b} &= \nu - \tau(y) - [\epsilon(y - \bar{y}) + \pi^e + g]\psi(\rho)y - \\
&- \psi'(\rho)yF_4 - \psi(\rho)\alpha F_2 + [\rho - \epsilon(y - \bar{y}) - \pi^e - g]b \equiv \\
&\equiv F_6.
\end{aligned}
$$

Fiscal and monetary policies mixture in our model

Regardless of properties of blue-marked functions, we have

$$F_3|_{\pi^e=\bar{\pi}} = 0,$$
$$F_4|_{\pi^e=\bar{\pi},y=\bar{y}} = 0,$$
$$F_5|_{\pi^e=\bar{\pi},b=\bar{b}} = 0.$$

$\bar{y}, \bar{\pi}, \bar{b}$ – target values of output, rate of inflation and bond to capital ratio prescribed by government and central bank, respectively.

Fiscal and monetary policies mixture in our model - normal equilibrium

An equilibrium

$$E = (d^*, y^*, \pi^{e*}, \rho^*, \nu^*, b^*) = (d^*, \bar{y}, \bar{\pi}, \rho^*, \nu^*, \bar{b})$$

is called *normal equilibrium*.

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\dot{b} &= \nu - \tau(y) - [\epsilon(y - \bar{y}) + \pi^e + g]\psi(\rho)y - \\
&- \psi'(\rho)yF_4 - \psi(\rho)\alpha F_2 + [\rho - \epsilon(y - \bar{y}) - \pi^e - g]b.
\end{aligned}
$$

$i(\rho, d)$ - nominal rate of interest applied to firm's private debt

The Equations (V1)

$$
\begin{aligned}
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\end{aligned}
$$

$\Phi(g)$ - adjustment cost function of investments

The Equations (V1)

$$
\begin{aligned}
\dot{d} &= \Phi(g) - s_f[r - i(\rho, d)d] - (g + \pi)d \equiv F_1, \\
\dot{y} &= \alpha[c + \Phi(g) + \nu - y] \equiv \alpha F_2, \ \ \alpha > 0 \\
\dot{\pi}^e &= \gamma[\xi(\bar{\pi} - \pi^e) + (1 - \xi)(\pi - \pi^e)] \equiv F_3, \\
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\dot{b} &= \nu - \tau(y) - [\epsilon(y - \bar{y}) + \pi^e + g]\psi(\rho)y - \\
&\quad - \psi'(\rho)yF_4 - \psi(\rho)\alpha F_2 + [\rho - \epsilon(y - \bar{y}) - \pi^e - g]b.
\end{aligned}
$$

g - rate of capital accumulation

The Equations (V1)

$$i(\rho, d) = \rho + i_1 d,$$

$$\Phi(g) = ag^2,$$

$$g = \frac{\kappa}{1 + e^{md + o(\rho - \pi^e) + n\beta y}}.$$

The Equations (V2)

$$\dot{d} = a\left(\frac{\kappa}{1+e^{q}}\right)^{2} - s_f[\beta y - (\rho + i_1 d)d] -$$

$$- \left[\frac{\kappa}{1+e^{q}} + \epsilon(y - \bar{y}) + \pi^{e}\right] d,$$

$$\dot{y} = \alpha\{(1 - s_1)[(1 - s_f\beta - \tau_1)y + T_0] +$$

$$+(1 - s_2)(\rho + i_1 d)d + (1 - s_3)\rho b +$$

$$+a\frac{\kappa}{1+e^{q}} + \nu - y\Big\},$$

$$\dot{\pi}^{e} = \gamma[\xi(\bar{\pi} - \pi^{e}) + (1 - \xi)\epsilon(y - \bar{y})],$$

The Equations (V2)

$$\dot{\rho} = \beta_1(\pi^e - \bar{\pi}) + (\beta_1\epsilon + \beta_2)(y - \bar{y}),$$

$$\dot{\nu} = \sigma[\theta(\bar{y} - y) + (1 - \theta)(\bar{b} - b)],$$

$$\dot{b} = \nu - \tau(y) - [\epsilon(y - \bar{y}) + \pi^e + \frac{\kappa}{1 + e^{md + o(\rho - \pi^e) + n\beta y}}]\psi(\rho)y -$$

$$-\psi'(\rho)yF_4 - \psi(\rho)\alpha F_2 + [\rho - \epsilon(y - \bar{y}) - \pi^e - \frac{\kappa}{1 + e^{md + o(\rho - \pi^e) + n\beta y}}]b$$

Independence of the normal equilibrium (NE) on some parameters

NE does not depend upon the following parameters

$$\alpha, \gamma, \sigma, \epsilon, \xi, \beta_1, \beta_2, \theta.$$

Hopf-Andronov bifurcation in our model

to show that HA bifurcation takes place in our model we follow the following standard steps

- to construct J-matrix at E
- to use Liu's criterion to J-matrix to choose a manifold $\mathcal{M}$ in parametric space at which J-matrix has a pair of pure imaginary and conjugated eigenvalues and four negative eigenvalues
- to transform the model to the form with Jordan linear approximation
- to transform the model to its partial normal form on invariant surface
- to construct the bifurcation equation

Economically relevant choice of the bifurcation parameter – credibility parameter ξ (credibility of targeting inflation by central bank)

$$
\begin{aligned}
\dot{d} &= \Phi(g) - s_f[r - i(\rho, d)d] - (g + \pi)d, \\
\dot{y} &= \alpha[c + \Phi(g) + \nu - y] \\
\dot{\pi}^e &= \gamma[\xi(\bar{\pi} - \pi^e) + (1 - \xi)(\pi - \pi^e)], \ \xi \in [0, 1], \\
\dot{\rho} &= \beta_1(\pi - \bar{\pi}) + \beta_2(y - \bar{y}), \\
\dot{\nu} &= \sigma[\theta(\bar{y} - y) + (1 - \theta)(\bar{b} - b)], \\
\dot{b} &= \nu - \tau(y) - [\epsilon(y - \bar{y}) + \pi^e + g]\psi(\rho)y - \\
&- \psi'(\rho)yF_4 - \psi(\rho)\alpha F_2 + [\rho - \epsilon(y - \bar{y}) - \pi^e - g]b
\end{aligned}
$$

Expectations

There should be a critical value ξ_0 such that

- above ξ_0 E is stable
- below ξ_0 E is unstable
- supercritical HA bifurcation takes place when crossing the value ξ_0

Bifurcation equation

Jordan form obtained for our system

$$
\begin{aligned}
\dot{d}_2 &= i\omega_0 d_2 + G_1(d_2, y_2, \pi_2^e, \rho_2, \nu_2, b_2, \xi),\\
\dot{y}_2 &= -i\omega_0 y_2 + G_2(d_2, y_2, \pi_2^e, \rho_2, \nu_2, b_2, \xi),\\
\dot{\pi}_2^e &= \lambda_3 \pi_2^e + G_3(d_2, y_2, \pi_2^e, \rho_2, \nu_2, b_2, \xi),\\
\dot{\rho}_2 &= \lambda_4 \rho_2 + G_4(d_2, y_2, \pi_2^e, \rho_2, \nu_2, b_2, \xi),\\
\dot{\nu}_2 &= \lambda_5 \nu_2 + G_5(d_2, y_2, \pi_2^e, \rho_2, \nu_2, b_2, \xi),\\
\dot{b}_2 &= \lambda_6 b_2 + G_6(d_2, y_2, \pi_2^e, \rho_2, \nu_2, b_2, \xi),
\end{aligned}
$$

Bifurcation equation

implies the bifurcation equation in the form

$$Ar^2 + B(\xi - \xi_0) = 0,$$

where

$$A = \mathrm{Re}\{\delta_2\},\ B = \mathrm{Re}\{\delta_1\}$$

where *finally*

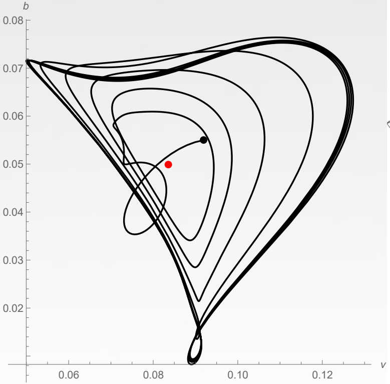
Bifurcation equation - resonant coefficients

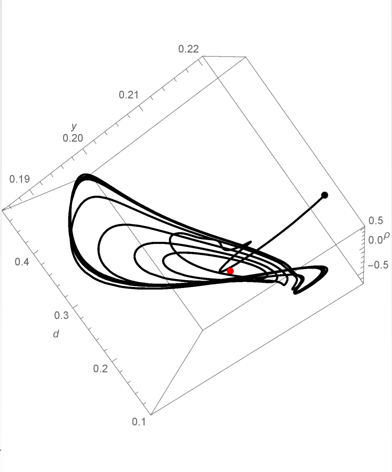
$$\delta_1 = \left.\frac{\partial^2 G_1}{\partial d_2 \partial \xi}\right|_{(E,\xi_0)}$$

Bifurcation equation - resonant coefficients

$\delta_2 =$ too long but explicit formula involving second and third derivatives of G

Example - close to Japanese economy in 2000 at $0.9 \times \xi_0$





Further research

- try to analyze global properties of our model
- try to investigate the appearance of chaos in our model – numerical simulations shows that cycle born due to HA bifurcation disappears at some value $\xi_- < \xi_0$ and is replaced by a strange attracting set

THANK YOU FOR YOUR KIND ATTENTION

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