

# Cycles in models of monetary and fiscal stabilization policies

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The background features a dark blue grid with several overlapping line charts in shades of purple, yellow, and green. Faint text labels like 'indexes - day, 5', 'indexes - day, 12', 'indexes - day, 55', and 'indexes - day' are visible, along with percentage values such as '61.81%', '50.00%', and '23.88%'. A large, semi-transparent teal circle is centered on the page, containing the title text.

# DYNAMICS IN MACROECONOMICS

The background of the slide features a complex overlay of financial data visualizations. It includes several line graphs with different colored lines (purple, green, yellow) and dashed trend lines. There are also candlestick-style bar charts. Faint text labels such as "indexes - day 5", "indexes - day 12", "day 55", and "indexes - day 200" are visible, along with numerical values like "61.81%", "38.19%", and "1925".

# THE MAIN GOAL

is to establish whether *stable* economical cycles are present in a class of models with flexible prices and mixture of fiscal and monetary policies.



# METHOD USED

We focus on transition between stable steady-state to unstable one with at least one stable limit cycle modeling business-cycles. So Hopf-Andronov theory is used.

# Introduction to model - used dynamical variables

We consider six-dimensional dynamical system with:

- $d$  - private debt/capital ratio
- $y$  - output/capital ratio
- $\pi^e$  - expected rate of prices inflation
- $\rho$  - nominal rate of interest of government bond
- $\nu$  - government expenditures/capital ratio
- $b$  - government bond/capital ratio

# The Equations (V1)

$$\dot{d} = \Phi(g) - s_f[r - i(\rho, d)d] - (g + \pi)d \equiv F_1,$$

$$\dot{y} = \alpha[c + \Phi(g) + \nu - y] \equiv \alpha F_2, \quad \alpha > 0$$

$$\dot{\pi}^e = \gamma[\xi(\bar{\pi} - \pi^e) + (1 - \xi)(\pi - \pi^e)] \equiv F_3,$$

$$\dot{\rho} = \beta_1(\pi - \bar{\pi}) + \beta_2(y - \bar{y}) \equiv F_4,$$

$$\dot{\nu} = \sigma[\theta(\bar{y} - y) + (1 - \theta)(\bar{b} - b)] \equiv F_5,$$

$$\begin{aligned} \dot{b} = & \nu - \tau(y) - [\epsilon(y - \bar{y}) + \pi^e + g]\psi(\rho)y - \\ & - \psi'(\rho)yF_4 - \psi(\rho)\alpha F_2 + [\rho - \epsilon(y - \bar{y}) - \pi^e - g]b \equiv \\ & \equiv F_6. \end{aligned}$$

# Fiscal and monetary policies mixture in our model

Regardless of properties of blue-marked functions, we have

$$F_3|_{\pi^e=\bar{\pi}} = 0,$$

$$F_4|_{\pi^e=\bar{\pi}, y=\bar{y}} = 0,$$

$$F_5|_{\pi^e=\bar{\pi}, b=\bar{b}} = 0.$$

$\bar{y}$ ,  $\bar{\pi}$ ,  $\bar{b}$  – target values of output, rate of inflation and bond to capital ratio prescribed by government and central bank, respectively.

# Fiscal and monetary policies mixture in our model - normal equilibrium

An equilibrium

$$E = (d^*, y^*, \pi^{e*}, \rho^*, \nu^*, b^*) = (d^*, \bar{y}, \bar{\pi}, \rho^*, \nu^*, \bar{b})$$

is called *normal equilibrium*.



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$i(\rho, d)$  - nominal rate of interest applied to firm's private debt

# The Equations (V1)

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$\Phi(g)$  - adjustment cost function of investments

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$g$  - rate of capital accumulation

# The Equations (V1)

$$i(\rho, d) = \rho + i_1 d,$$

$$\Phi(g) = ag^2,$$

$$g = \frac{\kappa}{1 + e^{md + o(\rho - \pi^e) + n\beta y}}.$$

# The Equations (V2)

$$\begin{aligned}\dot{d} &= a \left( \frac{\kappa \cdot \text{index} \cdot \text{day}}{1 + e^q} \right)^2 - s_f [\beta y - (\rho + i_1 d) d] - \\ &\quad - \left[ \frac{\kappa}{1 + e^q} + \epsilon(y - \bar{y}) + \pi^e \right] d, \\ \dot{y} &= \alpha \{ (1 - s_1) [(1 - s_f \beta - \tau_1) y + T_0] + \\ &\quad + (1 - s_2)(\rho + i_1 d) d + (1 - s_3) \rho b + \\ &\quad + a \frac{\kappa}{1 + e^q} + \nu - y \}, \\ \dot{\pi}^e &= \gamma [\xi(\bar{\pi} - \pi^e) + (1 - \xi)\epsilon(y - \bar{y})],\end{aligned}$$

# The Equations (V2)

$$\dot{\rho} = \beta_1(\pi^e - \bar{\pi}) + (\beta_1\epsilon + \beta_2)(y - \bar{y}),$$

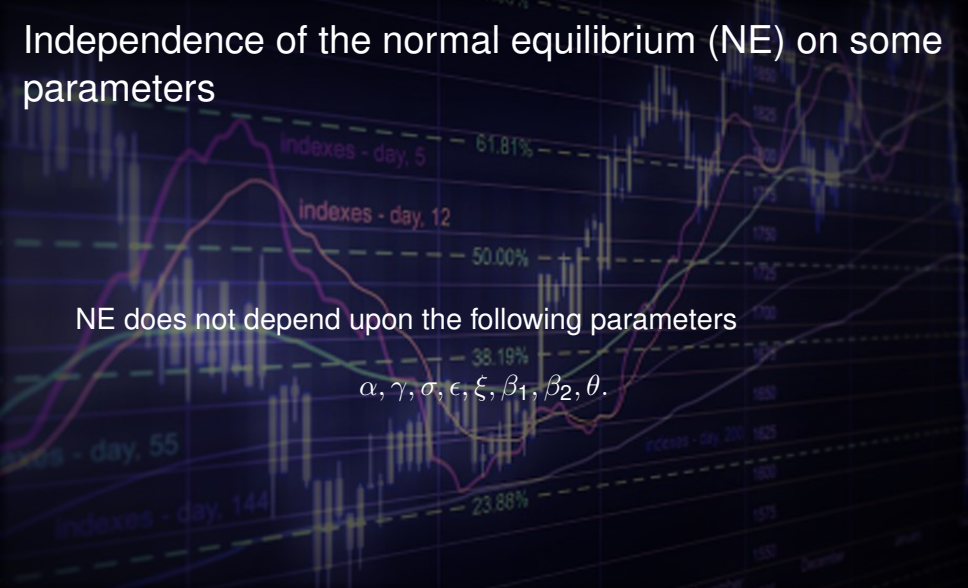
$$\dot{\nu} = \sigma[\theta(\bar{y} - y) + (1 - \theta)(\bar{b} - b)],$$

$$\dot{b} = \nu - \tau(y) - [\epsilon(y - \bar{y}) + \pi^e + \frac{\kappa}{1 + e^{md + o(\rho - \pi^e) + n\beta y}}]\psi(\rho)y - \\ - \psi'(\rho)yF_4 - \psi(\rho)\alpha F_2 + [\rho - \epsilon(y - \bar{y}) - \pi^e - \frac{\kappa}{1 + e^{md + o(\rho - \pi^e) + n\beta y}}]b$$

# Independence of the normal equilibrium (NE) on some parameters

NE does not depend upon the following parameters

$$\alpha, \gamma, \sigma, \epsilon, \xi, \beta_1, \beta_2, \theta.$$



# Hopf-Andronov bifurcation in our model

to show that HA bifurcation takes place in our model we follow the following standard steps

- to construct  $J$ -matrix at  $E$
- to use Liu's criterion to  $J$ -matrix to choose a manifold  $\mathcal{M}$  in parametric space at which  $J$ -matrix has a pair of pure imaginary and conjugated eigenvalues and four negative eigenvalues
- to transform the model to the form with Jordan linear approximation
- to transform the model to its partial normal form on invariant surface
- to construct the bifurcation equation



Economically relevant choice of the bifurcation parameter – credibility parameter  $\xi$  (credibility of targeting inflation by central bank)

$$\begin{aligned}\dot{d} &= \Phi(g) - s_f[r - i(\rho, d)d] - (g + \pi)d, \\ \dot{y} &= \alpha[c + \Phi(g) + \nu - y], \\ \dot{\pi}^e &= \gamma[\xi(\bar{\pi} - \pi^e) + (1 - \xi)(\pi - \pi^e)], \quad \xi \in [0, 1], \\ \dot{\rho} &= \beta_1(\pi - \bar{\pi}) + \beta_2(y - \bar{y}), \\ \dot{\nu} &= \sigma[\theta(\bar{y} - y) + (1 - \theta)(\bar{b} - b)], \\ \dot{b} &= \nu - \tau(y) - [\epsilon(y - \bar{y}) + \pi^e + g]\psi(\rho)y - \\ &\quad - \psi'(\rho)yF_4 - \psi(\rho)\alpha F_2 + [\rho - \epsilon(y - \bar{y}) - \pi^e - g]b\end{aligned}$$

# Expectations

There should be a critical value  $\xi_0$  such that

- above  $\xi_0$   $E$  is stable
- below  $\xi_0$   $E$  is unstable
- supercritical HA bifurcation takes place when crossing the value  $\xi_0$

# Bifurcation equation

Jordan form obtained for our system

$$\dot{d}_2 = i\omega_0 d_2 + G_1(d_2, y_2, \pi_2^e, \rho_2, \nu_2, b_2, \xi),$$

$$\dot{y}_2 = -i\omega_0 y_2 + G_2(d_2, y_2, \pi_2^e, \rho_2, \nu_2, b_2, \xi),$$

$$\dot{\pi}_2^e = \lambda_3 \pi_2^e + G_3(d_2, y_2, \pi_2^e, \rho_2, \nu_2, b_2, \xi),$$

$$\dot{\rho}_2 = \lambda_4 \rho_2 + G_4(d_2, y_2, \pi_2^e, \rho_2, \nu_2, b_2, \xi),$$

$$\dot{\nu}_2 = \lambda_5 \nu_2 + G_5(d_2, y_2, \pi_2^e, \rho_2, \nu_2, b_2, \xi),$$

$$\dot{b}_2 = \lambda_6 b_2 + G_6(d_2, y_2, \pi_2^e, \rho_2, \nu_2, b_2, \xi),$$

# Bifurcation equation

implies the bifurcation equation in the form

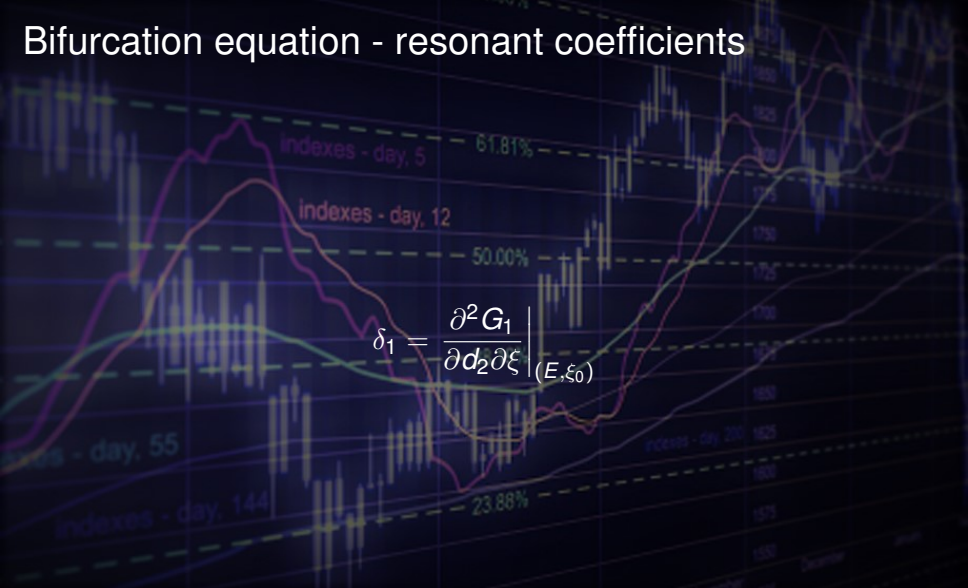
$$Ar^2 + B(\xi - \xi_0) = 0,$$

where

$$A = \operatorname{Re}\{\delta_2\}, \quad B = \operatorname{Re}\{\delta_1\}$$

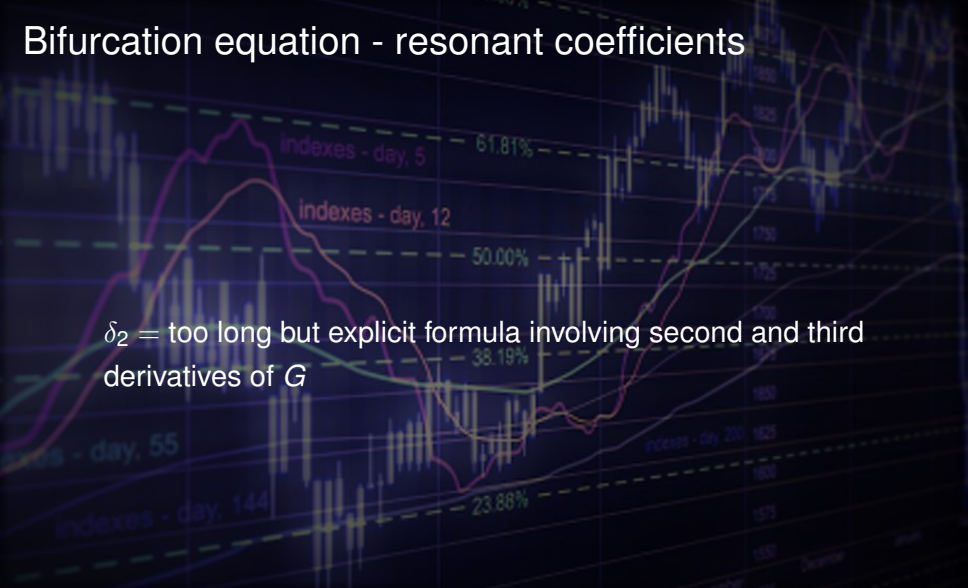
where *finally*

# Bifurcation equation - resonant coefficients

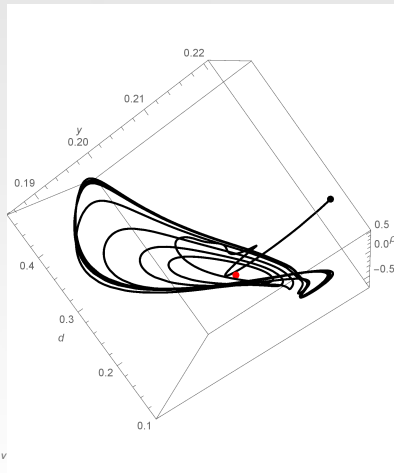
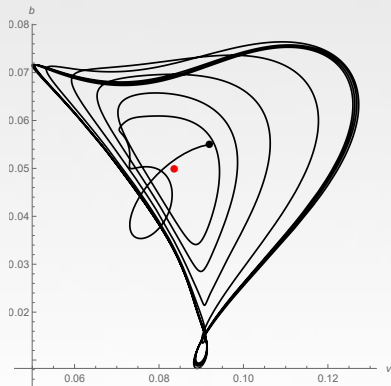


# Bifurcation equation - resonant coefficients

$\delta_2 =$  too long but explicit formula involving second and third derivatives of  $G$



Example - close to Japanese economy in 2000 at  $0.9 \times \xi_0$



## Further research

- try to analyze global properties of our model
- try to investigate the appearance of chaos in our model – numerical simulations shows that cycle born due to HA bifurcation disappears at some value  $\xi_- < \xi_0$  and is replaced by a strange attracting set



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THANK YOU FOR YOUR  
KIND ATTENTION

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THANK YOU