Regular and singular perturbation problems for infinite order system of differential equations with a small parameter

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In this paper, regular and singular perturbation problems for infinite order system of differential equations with a small parameter is considered. We can study the regularly perturbed Cauchy problem for a system of ordinary differential equations of infinite order with a small parameter μ and initial conditions of the form

$$\begin{cases} \dot{y}_i = F_i\left(y(t, G(\mu), \mu), t, \mu\right),\ i = 1, 2, \dots, \\ y_i(t_0, G(\mu), \mu) = G_i(\mu),\ i = 1, 2, \dots, \end{cases} \tag{1}$$

where $y(t, G(\mu), \mu) \in \mathbf{R}^\infty$, $F(y(t, G(\mu), \mu), t, \mu) \in \mathbf{R}^\infty$, $G(\mu) \in \mathbf{R}^\infty$ are infinite-dimensional functions and $t \in [t_0, t_0 + \Delta t]$ ($t_0 < t_0 + \Delta t \leq \infty$), $t \in T$, $T \in \mathbf{R}$; $\mu > 0$ is a small real parameter.

Then we can study the singularly perturbed Tikhonov-type Cauchy problems for systems of ordinary differential equations of infinite order with a small parameter μ and initial conditions of the form

$$\begin{cases} \dot{x}_i = f_i(x(t, g(\mu), \mu), y(t, G(\mu), \mu), t, \mu),\ i = 1, 2, \dots, n, \\ \mu^{s_i}\dot{y}_i = F_i(x(t, g(\mu), \mu), y(t, G(\mu), \mu), t, \mu),\ i = n+1, n+2, \dots, \\ x_i(t_0, g(\mu)) = g_i(\mu),\ i = 1, 2, \dots, n, \\ y_i(t_0, G(\mu)) = G_i(\mu),\ i = n+1, n+2, \dots, \end{cases} \tag{2}$$

where $x(t, g(\mu), \mu) \in \mathbf{R}^n$, $f(x(t, g(\mu), \mu), y(t, G(\mu), \mu), t, \mu) \in \mathbf{R}^n$, $g(\mu) \in \mathbf{R}^n$ are n-dimensional functions; $y(t, G(\mu), \mu) \in \mathbf{R}^\infty$, $F(y(t, G(\mu), \mu), t, \mu) \in \mathbf{R}^\infty$, $G(\mu) \in \mathbf{R}^\infty$ are infinite-dimensional functions and $t \in [t_0, t_0 + \Delta t]$ ($t_0 < t_0 + \Delta t \leq \infty$), $t \in T$, $T \in \mathbf{R}$; $\mu > 0$ is a small real parameter, $s_i \in \mathbf{N}$, $s_i > 0$ is a sequence of numbers.

The main results of this paper are concerned with the asymptotic behavior of solutions of the problems (1) and (2). The theorems of existence and uniqueness of solutions and the theorems on the continuous dependence on a parameter $\mu > 0$ are proved, asymptotic solutions are constructed and an estimate of the remainder terms of the asymptotic expansions are studied for these problems. These results may be applied to the queueing network problems.

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References

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