Delayed exchange of stabilities for a singularly perturbed initial value problem

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We consider a singularly perturbed initial value problem in the case of intersecting quasi stationary manifolds. The main results are concerned with the asymptotic behavior of solutions as the small parameter tends to zero.

Our results are related to the Tikhonov approach. The main condition for the validity of the Tikhonov theorem is that the quasi steady states be isolated and attractive. In applications, however, we often encounter the situation when two or more quasi steady states intersect. It involves the so called *exchange of stabilities*: the branches of the quasi steady states change from being attractive to being repelling (or conversely) across the intersection. The assumptions of the Tikhonov theorem fail to hold in the neighbourhood of the intersection but it is natural to expect that any solution that passes close to it follows the attractive branches of the quasi steady states on either side of the intersection. However, in many cases an unexpected behaviour of the solution is observed – it follows the attracting part of a quasi steady state and, having passed the intersection, it continues along the now repelling branch of the former quasi steady state for some prescribed time and only then jumps to the attracting part of the other quasi steady state. Such a behaviour we call the *delayed switch of stability*. We shall focus on the so called *backward bifurcation*, in which two of three quasi steady states intersect and exchange stabilities at the intersection.

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