Asymptotic solution of the linearized Korteweg-de Vries equation

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Consider the linearized one-dimensional Korteweg-de Vries equation with variable coefficients

$$\psi_t + (C(x,\, t)\psi)_x + h^2\psi_{xxx} = 0 \tag{1}$$

with small parameter h. For this equation we pose the initial problem with localized data

$$\psi|_{t=0} = V\left(\frac{x-\xi}{\mu}\right), \tag{2}$$

where another small parameter μ describes the localization of the initial function.

In the case of constant coefficient $C(x,\, t) \equiv C$ the Green function for (1) is known and it has the form of Airy function. The exact solution can be written in the form of the Fourier transform.

If the coefficint in (1) is variable then the solution for (1)-(2) has more complicated form. The localized function V can be represented with the help of the Maslov canonical operator [1]. This representation allows to impliment the full theory of the Maslov canonical operator [2] for constructing the asymptotic solution of the intial problem.

We study the asymptotic solution of the initial problem while $\mu \to 0$. One can determine the leading wave front using the ideas of the work [3] and construct the asymptotic of the wave near this front. This wave can be represented via Airy function and the formula is similar to the Green function for the problem with constant coefficient.

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