On solutions with prescribed number of zeros to a fourth-order regular Emden-Fowler type equation with negative potential

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We consider the Emden-Fowler type equation

$$y^{IV} - p(t, y, y', y'', y''')|y|^k \operatorname{sgn} y = 0, \tag{1}$$

where $k > 1$, $p(t, \xi_1, \xi_2, \xi_3, \xi_4)$ is a continuous function and at the same time it is Lipschitz continuous in $(\xi_1, \xi_2, \xi_3, \xi_4)$. In addition, the inequalities $0 < m \le p(t, \xi_1, \xi_2, \xi_3, \xi_4) \le M < +\infty$ hold for some m, M.

Theorem 1. *For any segment $[a, b]$ there exists a number N such that for any integer $S > N$ there exists a solution to (1) defined on $[a, b]$, vanishing at the points a and b, and having exactly S zeros on $[a, b]$.*

Remark 2. Theorem 1 follows from the classification of solutions to fourth-order Emden-Fowler type equation with constant coefficient (see [1], ch. **6, 7**; [2]).

The result on the existence of a solution with prescribed number of zeros to the regular Emden-Fowler type equation with positive potential is proved in [3](n=3,4), [4], and, for the singular case, in [5].

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References

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