State-dependent impulses and distributions in BVPs

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This is a common research with Jan Tomeček from Palacký University in Olomouc. We deal with problems where the evolution of systems is affected by rapid changes which are modelled by means of differential equations with impulses. Abrupt changes of solutions of impulsive problems imply that such solutions do not preserve the basic properties which are associated with non-impulsive problems. We focus our attention to problems with a finite number $m \in \mathbb{N}$ of impulses on the compact interval $[0, T] \subset \mathbb{R}$. Most papers deal with *fixed-time* impulses where the moments of impulses $0 < t_1 < t_2 < \cdots < t_m < T$ are fixed and known before. This is a special case of so called *state-dependent* impulses where the impulse moments depend on a solution of a differential equations and different solutions can have different moments of jumps. We present two ways of determining the impulse dependence on the solution:

- Let $\tau_1, \ldots, \tau_m$ be functionals defined on a suitable functional space X and having values in $(0, T)$. Then the impulse moments are given as

$$t_i = \tau_i(x) \in (0, T), \quad x \in X, \quad i = 1, \ldots, m.$$

- Let $\gamma_1, \ldots, \gamma_m$ be functions (barriers) defined on a suitable interval $[a, b] \subset \mathbb{R}$ and having values in $(0, T)$. Then the impulse moments are given as

$$t_i = \gamma_i(x(t_i)) \in (0, T), \quad x \in X, \quad i = 1, \ldots, m.$$

  In order to get the desired number of impulse points in this case it is necessary to impose additional conditions (transversality conditions) on $\gamma_1, \ldots, \gamma_m$.

Existence results for periodic, anti-periodic and Dirichlet problems for the second order differential equation with state-dependent impulses are discussed and a comparison with distributional equations is shown.

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