Turing-like phenomenon on a discrete space-time

Zdeněk Pospíšil

Brno, Czech Republic

We consider the coupled recurrences (difference equations)

$$u(t+\theta) = f\bigl(u(t), v(t)\bigr), \quad v(t+\theta) = g\bigl(u(t), v(t)\bigr),$$

possessing an asymptotically stable equilibrium (u^*, v^*). A particular choice of the nonlinear functions $f, g : \mathbb{R}_+^2 \to \mathbb{R}_+$ enables one to interpret these equations as a model of a two-component chemical reaction, of two populations interaction, of two ideologies competition, and the like.

Let $\mathcal{G} = (N, E)$ be a simple graph. A process of the reaction in a node followed by diffusion of components (dispersion of populations) on the graph $\mathcal{G}$, i. e. a random move of a particle (individual) from a node to a neighbour one, can be described by the discrete system

$$\left.\begin{aligned} x_i(t+1) &= (1-d_1) f\bigl(x_i(t), y_i(t)\bigr) + d_1 \sum_{\{i,j\}\in E} \frac{f\bigl(x_j(t), y_j(t)\bigr)}{\sigma_j}, \\ y_i(t+1) &= (1-d_2) g\bigl(x_i(t), y_i(t)\bigr) + d_2 \sum_{\{i,j\}\in E} \frac{g\bigl(x_j(t), y_j(t)\bigr)}{\sigma_j}, \end{aligned}\right. \quad i = 1, 2, \ldots, |N|. \tag{1}$$

Here $d_p \in]0,1]$ denotes the probability that a “particle” of the p-th component, $p = 1, 2$, remains during the unit time interval in the same node, $\sigma_j = \sum_{\{i,j\}\in E} 1$ is the degree of the j-th node.

If the adjacency matrix of the graph $\mathcal{G}$ is symmetric, then $x_i(t) \equiv u^*$, $y_i(t) \equiv v^*$ is a spatially homogeneous equilibrium of the system (1). The aims of the contribution consist in demonstration that this equilibrium need not to be stable and in presenting conditions for the instability. That is, in describing a discrete analogy of the well known diffusion-driven or Turing instability.

Acknowledgement

The research was motivated by attempts to model a diffusion dynamics of religious ideas and behavior forms, see www.gehir.phil.muni.cz, and it was supported by the grant MUNI/M/1867/2014.

2010 Mathematics Subject Classification: 39A30, 35K57, 00A69.