

Singular solutions of Protter problems for the wave equation

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We study boundary value problems for the nonhomogeneous wave equation in one time and three space variables, that are analogues of Darboux problems in the plane. Originally they were proposed by Murray H. Protter in the fifties, while studying BVPs for mixed type equations that model transonic flow phenomena. Initially, the expectation was that the Protter problem is classically solvable for very smooth right-hand side functions. However, soon it became clear that unlike the planar Darboux problem, its multi-dimensional analogue is not well posed. In fact, in the frame of classical solvability the Protter problem is not Fredholm, since it has infinite-dimensional cokernel. Alternatively, it is known that the unique generalized solution of the problem may have a strong power-type singularity. It is interesting that this singularity is isolated at the vertex O of the boundary light cone and does not propagate along the bicharacteristics, which makes this case different from the traditional case of propagation of singularity. We present some conditions on the smooth right-hand side functions that are sufficient for existence of a generalized solution and give a priori estimates for its growth at the point O . We construct an infinitely smooth right-hand side function, such that the generalized solution has an exponential singularity.

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References

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