How to determine the shape of the human cornea: a contribution from nonlinear analysis

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In this talk I will present some results, that have recently been obtained in [1, 2, 3, 4] and concern the following prescribed anisotropic mean curvature equation with Dirichlet boundary conditions:

$$\begin{cases} -\mathrm{div}\left(\dfrac{\nabla u}{\sqrt{1+|\nabla u|^2}}\right) = -au + \dfrac{b}{\sqrt{1+|\nabla u|^2}} & \text{in } \Omega, \\ u = 0 & \text{on } \partial\Omega. \end{cases} \tag{1}$$

Here, $\Omega \subset \mathbb{R}^N$ is a bounded regular domain and $a, b > 0$ are real parameters.

This quasilinear problem was introduced in [5, 6] in order to provide a mathematical model for describing the geometry of the human cornea.

I aim to show how various techniques of nonlinear analysis, from elementary to more sophisticated, can successfully be combined to derive a rather complete picture of the solvability patterns of problem (1), including existence, uniqueness, regularity, boundary behaviour, stability of solutions, as well as information on the structure of the solution set.

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References

[1] I. Coelho, C. Corsato and P. Omari, *A one-dimensional prescribed curvature equation modeling the corneal shape*, Bound. Value Probl. 2014, 2014:127.

[2] C. Corsato, C. De Coster and P. Omari, *Radially symmetric solutions of an anisotropic mean curvature equation modeling the corneal shape*, Discrete Contin. Dyn. Syst. 2015 Suppl. (2015), 297–303.

[3] C. Corsato, C. De Coster and P. Omari, *The Dirichlet problem for a prescribed anisotropic mean curvature equation: existence, uniqueness and regularity of solutions*, J. Differential Equations 260 (2016), 4572–4618.

[4] C. Corsato, C. De Coster, F. Obersnel, P. Omari and A. Soranzo, *A prescribed anisotropic mean curvature equation modeling the corneal shape: a paradigm of nonlinear analysis*, Discrete Contin. Dyn. Syst. Ser. S (2017), 1– 44 (in press).

[5] W. Okrasiński and Ł. Płociniczak, *A nonlinear mathematical model of the corneal shape*, Nonlinear Anal. Real World Appl. 13 (2012), 1498–1505.

[6] W. Okrasiński and Ł. Płociniczak, *Bessel function model for corneal topography*, Appl. Math. Comput., 223 (2011), 436–443.