An approximation to the minimum wave for Nicholson Blowflies Equation

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In this work [1], we will present the approximation of traveling waves solution propagated at minumum speeds $c_0(h)$ (critical case) of the delayed Nicholson Blowflies equation

$$u_t(t,x) = \Delta u(t,x) - \delta u(t,x) + pu(t-\hat{h},x)e^{-u(t-\hat{h},x)}, \quad u(t,x) \geq 0, \quad x \in \mathbb{R}^m, \tag{1}$$

where $\hat{h} \geq 0$ and the parameters p, δ satisfy $p/\delta \in (1,e]$. In order to do that, we construct a super and sub solution to (1). Also, by that construction, an alternative proof of existence of traveling waves moving at minimum speed is given. The main difficulty in this case is due by the multiplicity of the eigenvalue associated with the linearization about 0 equilibrium, where an adequate, and different to the super-critical case, sub-solution is required.

Our main theorem is

Theorem 1. *Let $p/\delta \in (1,e]$, $h \geq 0$ and $c = c_0(h)$. Then, Equation (1) possesses a traveling wave solution $u(t,x) = \phi(\nu \cdot x + ct)$. Moreover, its profile can be obtained as $\phi(t) = \lim_{n\to\infty} \phi_n(t)$, for all $t \in \mathbb{R}$, with defined by induction as follows:*

$$\phi_0(t) = \overline{\phi}(t) := \begin{cases} -k_1(t-t_0)e^{\lambda_1(t-t_0)}, & \text{if } t < 0, \\ \kappa - k_2 e^{(\mu_1-\epsilon_1)t}, & \text{if } t \geq 0, \end{cases}$$

with t_0, ϵ_1, k_1, k_2 defined by

$$t_0 := \tfrac{2}{\lambda_1}, \quad \epsilon_1 := \tfrac{-c_0+2\mu_1+\sqrt{(c_0-2\mu_1)^2+4\ln(e\delta/p)e^{-r\mu_1}}}{2}, \quad k_1 := \tfrac{\kappa\lambda_1(\epsilon_1-\mu_1)e^2}{\lambda_1+2(\epsilon_1-\mu_1)}, \quad k_2 := \tfrac{\kappa\lambda_1}{\lambda_1+2(\epsilon_1-\mu_1)}$$

and

$$\begin{aligned} \phi_{n+1}(t) := &\frac{p}{\delta(\alpha_2-\alpha_1)}\int_{-\infty}^{t} e^{\alpha_1(t-s)}\phi_n(s-r)e^{-\phi_n(s-r)}ds \\ &+ \frac{p}{\delta(\alpha_2-\alpha_1)}\int_{t}^{\infty} e^{\alpha_2(t-s)}\phi_n(s-r)e^{-\phi_n(s-r)}ds, \end{aligned}$$

for all $n \geq 0$.

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References

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