Systems of ordinary differential equations with nonlocal boundary conditions

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An existence result from the years 1960 (Krasnosel'skii) tells that if $f : [0,T] \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous and if there exists $R > 0$ such that

$$\text{either } \langle x, f(t,x)\rangle \geq 0 \quad \text{or} \quad \langle x, f(t,x)\rangle \leq 0, \quad \forall\, (t,x) \in [0,T] \times \partial B(R), \tag{1}$$

where $B(R) \subset \mathbb{R}^n$ denotes the open ball of center 0 and radius R, then the periodic problem

$$x' = f(t,x),\ x(0) = x(T) \tag{2}$$

has at least one solution such that $|x(t)| \leq R$ for all $t \in [0,T]$. The two results are equivalent, the second one being deduced from the first one through the change of variable $\tau = T - t$. Condition (1) has been generalized in several directions for problem (2).

Recently, in joint work with K. Szymańska-Dębowska, some extensions of those results have been obtained for nonlocal boundary conditions of the type

$$x(0) = \int_0^T dg(s)x(s) \quad \text{or} \quad x(T) = \int_0^T dg(s)x(s) \tag{3}$$

where g is a function with bounded variation from $[0,T]$ into diagonal $(n \times n)$-matrices, satisfying some conditions, and containing the periodic boundary conditions as special cases.

The situations (2) and (3) are compared with the use of counterexamples, showing the singularity of the periodic case.

This is a joint work with K. Szymańska-Dębowska.

2010 Mathematics Subject Classification: 34B10, 34B15, 47H11.

References

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