Radial solutions for nonlinear elliptic equation with nonlinear nonlocal boundary condtion

Igor Kossowski

Lodz, Poland

We consider the problem for the nonlinear elliptic partial differential equation in an annular domain

$$-\Delta u = f(|x|, u), \qquad u|_{\partial\Omega} = \int\limits_{\Omega} K(|x|, |y|)h(u(x))\,dx, \tag{1}$$

where $\Omega = B(0, b) \setminus \overline{B}(0, a) \subset \mathbb{R}^n$, $0 < a < b$.

The problem (1) was motivated by applications in a quasi-static theory of thermoelasticity. We look for radial solutions for (1). Using appropriate substitutions, the problem (1) was reduced to the second-order ordinary differential equation with a couple of two nonlinear nonlocal conditions. Using some fixed point theorem in a cone, we will prove the existence of at least one solution for the "reduced" problem.

2010 Mathematics Subject Classification: 34B10, 34B15.

References

[1] W. A. Day, *Extensions of a property of the heat equation to linear thermoelasticity and other theories*, *Quart. Appl. Math.*, 40(3) (**1982/83**), 319–330.

[2] W. A. Day, *Heat conduction within linear thermoelstacity*, *Springer Tracts in Natural Philosophy*, Volume 30, **1985**, Springer-Verlag, New York.

[3] K. Deng, *Comparison principle for some nonlocal problems*, *Quart. Appl. Math.*, 50 (**1992**), 517–522.

[4] D. Guo, V. Lakshmikantham, *Nonlinear problems in abstract cones*, *Academic Press*, Orlando, FL, **1988**.