Iterative techniques with initial time difference and computer realization for the initial value problem for Caputo fractional differential equations

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The main aim of the paper is to suggest some algorithms and to use them in an appropriate computer environment to solve approximately the initial value problem for scalar nonlinear Caputo fractional differential equations on a finite interval:

$$ {}^{c}_{t_0}D^{q}_{t}x(t) = f(t, x(t)) \quad \text{for } t \in [t_0, t_0 + T], \qquad x(t_0) = x_0 $$

where $x \in \mathbb{R}$, $q \in (0, 1)$, ${}^{c}_{t_0}D^{q}_{t}x(t)$ is the Caputo fractional derivative, the constants $T > 0$, $t_0 \in \mathbb{R}$ are given.

The base of the algorithms is the definition of mild lower solutions and mild upper solutions, given in the paper. Since sometimes it is easier to construct mild lower/upper solutions on intervals different than the given one, we consider the case of so called initial time difference. In the case of fractional differential equations, differently than the case of ordinary derivatives, the change of the initial time point has a huge influence not only on the interval of considerations but also on the fractional derivative. It requires totally different approach for constructing successive approximations. In this paper the suggested scheme does not use Mittag-Leffler functions. This make the practical application of the suggested algorithms easier. Every next approximation is obtained by an integral of a nonlinear function of the previous approximations. This makes the application of the suggested procedure easily which is shown in the paper by the computer applications to particular examples. Also, the suggested algorithms are compared on examples with the existing and published ones. The advantages of the given algorithms and their applications are demonstrated.

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