Protter-Morawetz problem for (3+1)-D equations of Keldysh type

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For $m \in \mathbb{R}$, $0 < m < 2$ we study a four-dimensional boundary value problem for nonhomogeneous mixed-type equations of the second kind

$$L_m[u] \equiv u_{x_1x_1} + u_{x_2x_2} + u_{x_3x_3} - (t^m u_t)_t = f(x,t), \tag{1}$$

expressed in Cartesian coordinates $(x,t) = (x_1, x_2, x_3, t)$ n the simply connected region

$$\Omega_m := \left\{ (x,t): \quad t > 0, \quad \frac{2}{2-m} t^{\frac{2-m}{2}} < |x| < 1 - \frac{2}{2-m} t^{\frac{2-m}{2}} \right\}.$$

Ω_m is bounded by the ball $\Sigma_0 := \{t = 0,\ |x| < 1\}$, centered at the origin O and by two characteristic surfaces of equation (1): $\Sigma_1^m := \left\{ t > 0,\ |x| = 1 - \frac{2}{2-m} t^{\frac{2-m}{2}} \right\}$, $\Sigma_2^m := \left\{ t > 0,\ |x| = \frac{2}{2-m} t^{\frac{2-m}{2}} \right\}$.

We consider the following problem:

Problem PK. Find a solution to equation (1) in Ω_m which satisfies the boundary conditions

$$u|_{\Sigma_1^m} = 0, \qquad t^m u_t \to 0 \text{ as } t \to +0.$$

The problem PK is an analogue of the Protter-Morawetz multidimensional problem for Tricomi-type equations formulated by M. Protter in connection with the classical Guderley-Morawetz plane problem that models transonic flow phenomena.

In this paper it is shown that problem PK is not well-posed in frame of classical solvability, since it has infinite-dimensional co-kernel. A notion of a generalized solution with possible singularity at point O is given. Results for existence and uniqueness of such solution are obtained [1]. Further, there are presented orthogonality conditions on the right-hand side function $f(x,t)$, which are necessary and sufficient for existence of generalized solution with fixed order of singularity [2].

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References

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