On various types of asymptotic behavior of oscillating solutions to second-order Emden-Fowler type differential equations

Kseniya Dulina

Moscow, Russia

Consider the second-order Emden-Fowler type differential equation

$$y'' + p\big(x, y, y'\big)|y|^k \operatorname{sgn} y = 0, \quad k > 0,\ k \neq 1, \tag{1}$$

where the function $p(x, u, v)$ defined on $\mathbb{R} \times \mathbb{R}^2$ is positive, continuous in x, Lipschitz continuous in u, v and satisfies the inequalities

$$0 < m \le p(x, u, v) \le M < +\infty. \tag{2}$$

Asymptotic behavior of nontrivial maximally extended solutions to (1) is investigated by using methods of [1]. The estimates obtained in [2] are improved. I. T. Kiguradze and T. A. Chanturia in [3] proved that if $p = p(x)$ is a positive locally integrable function of locally bounded variation, then both for regular $(k > 1)$ and singular $(0 < k < 1)$ nonlinearities, any nontrivial maximally extended to the right solution to (1) is proper, i. e. is defined in a neighborhood of $+\infty$. For $k > 1$ an example is given of a continuous function $p = p(x)$ satisfying inequalities (2) such that there exists a solution to (1) with a resonance asymptote $x = x^*$ ($\overline{\lim}_{x\to x^*} y(x) = +\infty$, $\underline{\lim}_{x\to x^*} y(x) = -\infty$), i. e. a non-proper solution. Moreover, sufficient conditions on the function $p = p(x)$ are obtained under which all nontrivial maximally extended solutions are defined on the whole axis. Asymptotic behavior of solutions is investigated in both cases when the conditions are and are not satisfied.

Theorem 1. *Suppose $k > 1$, $p = p(x)$ is a continuous function of globally bounded variation satisfying inequalities* (2)*. Then for any nontrivial maximally extended solution $y(x)$ to* (1) *the following finite positive limits exist:* $\lim_{j\to\pm\infty} |y'(x_j)|$, $\lim_{j\to\pm\infty} |y(x'_j)|$, *and* $\lim_{j\to\pm\infty}(x_{j+1} - x_j)$.

Remark 2. An example of a continuous function $p(x) > 0$ is given such that there exists an unbounded solution defined on $\mathbb{R}$. An example of a continuous function $p(x) > 0$ is given such that there exists a nontrivial proper oscillating solution tending at infinity to zero with first derivative.

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References

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