Convergence to a new profile of travelling front in Fisher's population genetics model

Pavel Drábek

Pilsen, Czech Republic

We consider the semilinear Fisher-Kolmogorov-Petrovski-Piscounov equation for the advance of an advantageous gene in biology:

$$\begin{cases} \dfrac{\partial u}{\partial t} - \dfrac{\partial^2 u}{\partial x^2} = f(u) & \text{for } (x,t) \in \mathbb{R} \times \mathbb{R}_+ ; \\ u(x,0) = u_0(x) & \text{for } x \in \mathbb{R} . \end{cases}$$

In contrast with previous works on this topic, we relax the differentiability hypothesis on f to being only Hölder-continuous and "one-sided" Lipschitz-continuous (i.e., $s \mapsto f(s) - Ls \colon \mathbb{R} \to \mathbb{R}$ is monotone decreasing, for some constant $L \in \mathbb{R}_+$). In particular, our hypotheses allow for the singular derivatives

$$f'(0) = \lim_{s \to 0} \frac{f(s)}{s} = -\infty \quad \text{and} \quad f'(1) = \lim_{s \to 1} \frac{f(s)}{s-1} = -\infty .$$

This type of reaction function f has been studied extensively in biological models of various kinds of generalized logistic growth.

The fact that reaction function f is not smooth allows for the introduction of travelling waves with a new profile. We study existence and uniqueness of this new profile, as well as a long-time asymptotic behavior of the solutions of the Cauchy problem to a travelling wave with this profile. Presented results are based on joint paper with P. Takáč entitled "Convergence to travelling waves in Fisher's population genetics model with a non-Lipschitzian reaction term" and published online in *J. Math. Biol.* on February 14, 2017.

2010 Mathematics Subject Classification: 35Q92, 92D10, 35K91, 35K58.