Applications of critical points results to existence and multiplicity of solutions for elliptic problems with variable exponent

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A collection of results concerning the existence and multiplicity of solutions for some elliptic problems involving the $p(\cdot)$-Laplacian is presented. The variable exponent p is supposed continuous and defined on $\bar{\Omega}$ where Ω is an open bounded subset of $\mathbb{R}^N$. The problems taken into account involve equations of the following kinds

$$-\Delta_{p(x)}u = \lambda f(x,u)$$

or

$$-\Delta_{p(x)}u + a(x)|u|^{p(x)-2}u = \lambda f(x,u)$$

with Dirichlet or Neumann boundary conditions.

For such problems the existence of at least three weak solutions, of a non trivial weak solution and of infinitely many weak solution will be established even in presence of a discontinuous non linear term and by using variational techniques. The results, obtained under opportune growth conditions on non linear term, concern the case

$$p^- := \inf_{x\in\Omega} p(x) > N$$

and the more general case

$$1 < p^- < p^+ := \sup_{x\in\Omega} p(x) < +\infty \cdot$$

However a precise interval of parameter λ for which the problems admit weak solutions is provided.

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