On a resolvent approach in a mixed problem for the wave equation on a graph

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We consider the simplest geometric graph consisting of two ring edges that touch at a point (at the node of the graph). Parametrizing each edge by the interval $[0,1]$, we study the following mixed problem for the wave equation on this graph:

$$\frac{\partial^2 u_j(x,t)}{\partial t^2} = \frac{\partial^2 u_j(x,t)}{\partial x^2} - q_j(x)u_j(x,t), \quad x \in [0,1],\ t \in (-\infty,+\infty), \quad (j=1,2), \tag{1}$$

$$u_1(0,t) = u_1(1,t) = u_2(0,t) = u_2(1,t), \tag{2}$$

$$u'_{1x}(0,t) - u'_{1x}(1,t) + u'_{2x}(0,t) - u'_{2x}(1,t) = 0, \tag{3}$$

$$u_1(x,0) = \varphi_1(x), \quad u_2(x,0) = \varphi_2(x), \quad u'_{1t}(x,0) = u'_{2t}(x,0) = 0 \tag{4}$$

(conditions (2), (3) are generated by the structure of the graph).

Based on the resolvent approach in the Fourier method and the Krylov convergence acceleration trick for Fourier series (see, [1] and the bibliography therein), we obtain a classical solution of this problem under minimal constraints on the initial condition. Note that no refined asymptotic formulas for the eigenvalues and any information on the eigenfunctions is employed.

The following result was obtained in [2]:

Theorem 1. *If $q_j(x) \in C[0,1]$ are complex-valued, $\varphi_j(x) \in C^2[0,1]$ and are complex-valued, $\varphi_1(0) = \varphi_1(1) = \varphi_2(0) = \varphi_2(1)$, $\varphi'_1(0) - \varphi'_1(1) + \varphi'_2(0) - \varphi'_2(1) = 0$, $\varphi''_1(0) = \varphi''_1(1) = \varphi''_2(0) = \varphi''_2(1)$, then the formal solution by Fourier method is a classical solution of problem* (1)–(4).

Moreover, the resolvent approach enables us to obtain a generalized solution of the problem (1)–(4) in the case of a summable potential using methods from [3].

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References

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