Approximate viability result for fractional differential inclusions in abstract Banach spaces

Mohamed Bezziou, Omar Benniche

Khemis miliana, Algeria

We consider fractional differential inclusions having the form

$$D^{\alpha}y(t) \in F(t, y(t)),\, y(\tau) = x, \tag{1}$$

where $F : I \times X \to X$ is a given set-valued function, $I \subset \mathbb{R}$ is an interval, X is a Banach space and $\alpha \in (0,1)$. Here and thereafter D^{α} stands for the Caputo fractional derivative of order α defined by:

$$D^{\alpha}y(t) = \frac{1}{\Gamma(1-\alpha)} \int_{\tau}^{t} (t-s)^{-\alpha} y'(s)ds,\, t \in (\tau, T).$$

Let $\mathbb{B}$ be the closed unit ball of X, that is $\mathbb{B} = \{x,\, \|x\| \leq 1\}$.

Definition 1. By an ε-solution of (1) on $[\tau, T] \subset I$ we mean an absolutely continuous function $y : [\tau, T] \to X$ satisfying $D^{\alpha}y(t) \in F(t, y(t) + \varepsilon\mathbb{B})$ for a.e. $t \in [\tau, T]$.

Let $G : I \rightsquigarrow X$ be a given set-valued function and let $\mathcal{K}$ be the graph of G.

Definition 2. We say that $\mathcal{K}$ is approximate viable with respect to (1), if for any $(\tau, x) \in \mathcal{K}$, there exists $T > \tau$ such that $[\tau, T] \subset I$ and for any $\varepsilon > 0$, there exist a function $\sigma : [\tau, T] \to [\tau, T]$, satisfying $t - \varepsilon \leq \sigma(t) \leq t$ for each $t \in [\tau, T]$ and an ε-solution $y : [\tau, T] \to X$ of (1) with $y(\tau) = x$ and

$$\text{dist}(y(t); G(\sigma(t))) \leq \varepsilon,$$

for all $t \in [\tau, T]$.

Here we provide sufficient and necessary conditions for the graph $\mathcal{K}$ to be approximate viable with respect to (1).

2010 Mathematics Subject Classification: 34C25, 34A60, 49J21.

References

[1] O. Benniche, O. Carja. Viability for quasi-autonomous semilinear evolution inclusions, *Mediterr.J. Math*, 13:4187–4210, 2016.

[2] O. Benniche, O. Carja and S. Djebali. Approximate Viability for Nonlinear Evolution Inclusions with Application to Controllability, Ann. Acad. Rom. Sci., 8:96–112, 2016.