A global estimates for solutions to quasi-autonomous differential equations in abstract Banach spaces

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We consider differential equations having the form

$$y'(t) = Ay(t)) + f(t, y(t)),\ y(t_0) = x_0, \tag{1}$$

where $f : I \times X \to X$ is a given function, $I \subset \mathbb{R}$ is an interval, X is a Banach space and $A : \overline{D(A)} \subset X \to X$ is an m–dissipative operator (possibly multi–valued) generating a nonlinear semigroup $S(t) : \overline{D(A)} \to \overline{D(A)}$, $t \geq 0$.

Definition 1. By an (integral) solution of (1) on $[t_0, T] \subset I$, we mean a continuous function $y : [t_0, T] \to X$ satisfying $y(t_0) = x_0$ and for every $u \in D(A)$ and $t_0 \leq \tau < t \leq T$ the following inequality holds

$$|y(t) - u| \leq |x_0 - u| + \int_\tau^t [y(s) - u, f(s) + Au]_+ ds.$$

We use here the right directional derivative of the norm,

$$[x, u]_+ = \lim_{h \to 0^+} \frac{\|x + hu\| - \|x\|}{h}.$$

Theorem 2. *Let X be a Banach space and let $A : \overline{D(A)} \subset X \to X$ be an m–dissipative operator which generates a compact C_0-semigroup, $S(t) : \overline{D(A)} \to \overline{D(A)}$, $t \geq 0$ and f is a continuous function, mapping bounded subsets of X into bounded subsets in X. Suppose that there exists $c > 0$ such that*

$$[x, f(x)]_+ \leq c(1 + k\|x\|^{-2})\|x\|;$$

for each $x \in X$. Then, for each $x_0 \in X \setminus \{0\}$ there exists $y : [0, +\infty) \to X$, a global solution to (1), that satisfies

$$\|y(t)\| \leq \|x_0\| e^{\theta t}$$

for every $t \geq 0$, for some $\theta > 0$.

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References

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