On asymptotic properties of blow-up and Kneser solutions to higher-order Emden-Fowler type differential equations

Irina Astashova

Moscow, Russia

Consider the equation

$$y^{(n)} = p(x, y, y', \dots, y^{(n-1)})|y|^k \operatorname{sign} y, \;\; n > 4, \;\; k > 1. \tag{1}$$

A new result is proved on asymptotic behavior of blow-up and Kneser (see [1, Definition 13.1]) solutions to this equation.

Theorem 1. *Suppose* $p \in C(\mathbf{R}^{n+1}) \bigcap Lip_{y_0,\dots,y_{n-1}}(\mathbf{R}^n)$ *and* $p \to p_0 > 0$ *as* $x \to x^*, y_0 \to \infty, \dots, y_{n-1} \to \infty$. *Then for any integer* $n > 4$ *there exists* $K > 1$ *such that for any real* $k \in (1, K)$, *any solution to equation* (1) *tending to* $+\infty$ *as* $x \to x^* - 0$ *has power-law asymptotic behavior, namely* $y(x) = C(x^* - x)^{-\alpha}(1 + o(1))$ *with*

$$\alpha = \frac{n}{k-1}, \quad C^{k-1} = \frac{1}{p_0} \prod_{j=0}^{n-1} (j + \alpha)\,. \tag{2}$$

Theorem 2. *Suppose* $p \in C(\mathbf{R}^{n+1}) \bigcap Lip_{y_0,\dots,y_{n-1}}(\mathbf{R}^n)$ *and* $(-1)^n p \to p_0 > 0$ *as* $x \to \infty$, $y_0 \to 0, \dots, y_{n-1} \to 0$. *Then for any integer* $n > 4$ *there exists* $K > 1$ *such that all Kneser solutions to equation* (1) *with any real* $k \in (1, K)$ *tend to zero with power-law asymptotic behavior, namely* $y(x) = C(x - x^*)^{-\alpha}(1 + o(1))$, $x \to \infty$, *with some* x^* *and* α, C *given by* (2).

Earlier it was proved that for $n = 3, 4$ all blow-up and Kneser solutions to equation (1) have the power-law asymptotic behavior (see [2]). It was also proved for equation (1) with $(-1)^n p \equiv p_0 > 0$ for sufficiently large n (see [3]) and for $n = 12, 13, 14$ (see [4]) that there exists $k > 1$ such that equation (1) has a solution with non-power-law behavior, namely $y(x) = (x - x^*)^{-\alpha} \, h(\log (x - x^*))$, where h is a positive periodic non-constant function on $\mathbf{R}$. For blow-up solutions see also [4, 5].

2010 Mathematics Subject Classification: 34C99.

References

[1] Kiguradze I. T., Chanturia T. A. Asymptotic Properties of Solutions of Nonautonomous Ordinary Differential Equations. Kluver Academic Publishers, Dordreht-Boston-London. (1993).

[2] I. Astashova. Qualitative properties of solutions to quasilinear ordinary differential equations (in Russian), in: I. V. Astashova (ed.), *Qualitative Properties of Solutions to Differential Equations and Related Topics of Spectral Analysis: scientific edition*, UNITY-DANA, Moscow (2012), 22–290.

[3] V. A. Kozlov. On Kneser solutions of higher order nonlinear ordinary differential equations, *Ark. Mat.* **37** (1999), No 2, 305–322.

[4] I. Astashova. On power and non-power asymptotic behavior of positive solutions to Emden-Fowler type higher-order equations, *Advances in Difference Equations. SpringerOpen Journal.* (2013), No 2013:220. 1–15.

[5] I. Astashova. On Kiguradze's problem on power-law asymptotic behavior of blow-up solutions to Emden–Fowler type differential equations, *Georgian Mathematical Journal,* 2017, 24(2), 185–191.